RANDOMIZING TIES IN A SIGN RADAR DETECTOR

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ABSTRACT

A general formulation to consider the effects of typical randomization methods (RMs) for a digital application of Generalized Sign Test (GST) detector {1}{2}{3} in Radar is introduced. A first approximation leads us to some basic restrictions to be imposed to RMs. Introducing them, when the approximation is acceptable, our formulation allows to evaluate easily the false alarm and detection probabilities (P_{FA} and P_D) obtainable with the use of each RM in fuction of the quantizing step (q) of the video samples, and, then, to select the most appropriate among them. Besides this, by considering the values of P_{FA} and P_D with respect to continuous situations, we can determine the maximum q to obtain small enough variations due to quantization (which has parametric effects). In such a way, a maximum dynamic range and a basically nonparametric behaviour are achieved. An example illustrates the application of the theory.

INTRODUCTION

The possibility of appearing clutter and/or countermeasures in Radar detection justifies the search for robustness in its schemes. After first approaches (CAG, CFAR), nonparametric (NP) and adaptive systems are now considered.

A NP test that has been found useful in 2D Radar detection is GST {1}{2}{3}. It is simple and distribution-free (DF) when independent, indentically distributed (IID) video samples (only noise) are present in each azimuth and there is azimuthal independence. This test works in the form

$$Z_{i,j} = \sum_{i'(N)} \sum_{\substack{j'(M) \\ j' \neq 0}} u(X_{i+i',j} - X_{i+i',j+j'}) \underset{H_0}{\overset{H_1}{\gtrless}} T \tag{1}$$

where i,i' and j,j' are azimuth and range indexes, respectively, X indicates video samples, u is the unit step function, N is the number of integrated pulses (transmitted pulses per antenna beamwidth), T is a threshold and H_1/H_0 are the target/no-target hypotheses. GST is a member of the rank-order tests {4} since it uses the rank statistics

$$R_{i+i',j} = \sum_{\substack{j'(M) \\ j' \neq 0}} u(X_{i+i',j} - X_{i+i',j+j'}) \tag{2}$$

There are other NP tests applied to Radar that can be considered as (partially) rank-binary quantized versions of GST {5}{6} . 2D NP test based on rank-order statistics show more complexity, require that the equidistribution condition be maintained in azimuth and do not offer important advantages {2}{7} .

I. QUANTIZATION, TIES, RANDOMIZATION AND DYNAMIC RANGE

It is usual to implement (1) with quantized video samples for simplicity; when a digital Moving Target Indicator (MTI) is present, this implementation is forced. A previous quantization simplifies also the approximation to linear or square-law envelope detectors {8} to obtain video samples. We will not consider the effects of MTI and envelope detection approximations since we only need reasonably accurate results.

Then, ties ($X_{i+i',j} = X_{i+i',j+j'}$) appear, and a decision becomes necessary to consider them. This has been a classical problem in Statistics (see {4} , for instance). There are many possible solutions, most of which can be considered as RMs, i.e., decision rules based on an additional experiment.

The selection of RM has a remarkable interest: it is necessary that the effect of ties be negligible even when the quantizing step implicates a coarse quantization (practically, this corresponds to situations in which there is only thermal noise as background) as long as possible, in order to avoid the loss of NP character maintaining the widest possible dynamic range (to prevent clutter and/or countermeasures saturation of the receiver). Then, an analysis of different RMs is desirable.

It is possible to randomize in accordance with several parameters. We will consider here two of them:

* the number of ties in each rank calculation, C (a random variable);
* the quantizing level k of the checked sample $X_{i+i',j}$.

Other RMs can be easily included in the corresponding formulation.

To analyze RMs, we can write

$$\Pr\left[\tilde{R}_{i+i',j} = l \,|\, \tilde{X}_{i+i',j} = n_k, S, C = c\right] = \Pr\left[\tilde{R}^{*(M-c)}_{i+i',j} = l \,|\, \tilde{X}_{i+i',j} = n_k, S\right] \circledast A_{k,c}(l)$$
$$l = 0,\ldots,M \qquad (3)$$

where $A_{k,c}(l)$ (the randomization function) is the probability of adding l to the (partial) rank obtained from no tied comparisons, $\tilde{R}^{*(M-c)}_{i+i',j}$; $\{n_k\}$, $k = 0,\ldots,Q-1$, are the quantizing levels; S is the (averaged) mean power of the signal (S = 0 corresponds to H_0); $\circledast$ indicates discrete convolution; $\sim$ is introduced to mark the quantization effects, and we have assumed that a target can only appear in the checked range (j). From (3),

$$\Pr\left[\tilde{R}_{i+i',j} = 1 \mid S\right] = \sum_{k=0}^{Q-1} p_k(S) \sum_{c=0}^{M} \binom{M}{c} p_k^c(0) \left[1 - p_k(0)\right]^{M-c} \{\Pr\left[\tilde{R}^{*(M-c)}_{i+i'j} = 1 \mid \tilde{X}_{i+i',j} = n_k, S\right] \circledast A_{k,c}(1)\}$$

$$1 = 0,\ldots,M \tag{4}$$

where $p_k(S) = F_s(c_{k+1}) - F_s(c_k)$ is the probability of $X_{i+i',j}$ quantized by n_k; f_s/F_s being the pdf/PDF of the video samples corresponding to a target of strength S, and $\{c_k\}$, $k = 0,\ldots,Q$ ($c_0 = 0$, $c_Q = \infty$), being the comparison levels of quantization.

II. A FIRST APPROXIMATION AND DISCUSSION

Expression (4) would require numerical manipulation to obtain exact results. The alternative of Monte Carlo simulation is not practical bacause Importance Sampling (IS) technique {3}{9}{10} connot be applied: it alters the production of ties. But, since proposed RMs should be applicable independently of N and Gaussian approximations for $Z_{i,j}$ are acceptable when N is high, we can admit these approximations to discuss RMs with a certain generality. (The validity of Gaussian hypotheses in the only noise cases can be checked in the corresponding continuous situations as a first guide).

The mean and variance of $\tilde{R}_{i+i',j}$ suffice to characterize $Z_{i,j}$ when we assume Gaussianity and azimuthal independence. It is not difficult to compute the mean value

$$\tilde{m}(S) = M \sum_{k=0}^{Q-1} p_k(S) \left[F_0(c_k) + \bar{\bar{\mu}}(k)\right] \tag{5}$$

where

$$\bar{\bar{\mu}}(k) = (1/M) \sum_{c=0}^{M} \mu(k,c) \binom{M}{c} p_k^{\,c}(0) \left[1-p_k(0)\right]^{M-c} = (1/M) \sum_{c=0}^{M} \sum_{1=0}^{c} 1A_{k,c}(1) \binom{M}{c} p_k^{\,c}(0)\left[1-p_k(0)\right]^{M-c} \tag{6}$$

In a continuous case, it is easy to obtain {11}

$$m(S) = M\int_0^\infty F_0(X)\, dF_s(X) \tag{7}$$

Starting from the expression corresponding to P_{FA} in a continuous case (which is a forced guide, since when clutter and/or countermeasures be present the video power will increase and quantizing and ties become uninportant), one finds in a digitalized application

$$\tilde{P}_{FA} = 1 - \phi\{\sqrt{N}\left[m(0) - \tilde{m}(0)\right] / \tilde{\sigma}(0) + \left[\sigma(0)/\tilde{\sigma}(0)\right] \phi^{-1}(1 - P_{FA})\} \tag{8}$$

where $\sigma(0)$, $\tilde{\sigma}(0)$, are rms values of $R_{i+i',j}$, $\tilde{R}_{i+i',j}$, respectively, ϕ is the Gaussian PDF and ϕ^{-1} is its inverse function. Then, if we select:

* $m(0) > m(0)$, $\tilde{P}_{FA}$ will increase without bound;
* $m(0) < m(0)$, $\tilde{P}_{FA}$ will decrease with N towards zero, but this would implicate a progressive decrease of detectability.

Thus, it seems reasonable to choose

$$\tilde{m}(0) = m(0) = \int_0^\infty F_0(X)\, dF_0(X) = 1/2 \tag{9}$$

prefixed error bound considering that

$$\sum_{k=K+1}^{Q-1} p_k(S)\left[F_0(c_k) + p_k(0)/2\right]^2 < \sum_{k=K+1}^{Q-1} p_k(S)\left[F_0(c_k) + p_k(0)/2\right] < \sum_{k=K+1}^{Q-1} p_k(S) \simeq 1 - F_s(c_{K+1}) \tag{13}$$

and that the $\gamma(S)$ contribution will not be important at this step of calculation if the error bound is low.

IV. AN EXAMPLE

A simple case in which numerical integrations are not needed to evaluate means and variances is the following: they are assumed a (fluctuating) Swerling II target {12} , a Gaussian and white background noise and a linear envelope detector; being the quantic values $\{c_k\} = \{0, q/2, 3q/2 \ldots, \infty\}$ and $\{n_k\} = \{0, q, 2q, \ldots, (Q-1)q\}$ (that implicate a linear quantization before envelope detection). S represents mean signal power averaged over all fluctuations; if we assume unity noise power, S will also indicate signal-to-noise ratio.

It is well known that, under these conditions, video sample PDS is

$$F_s(X) = \{1 - \exp\left[-X^2/2(1+S)\right]\} \ u(X) \tag{14}$$

and, from this,

$$p_k(S) = \begin{cases} 1 - \exp\left[-q^2/8(1+S)\right], & k = 0 \\ \exp\left[-(2k-1)^2q^2/8(1+S)\right] - \exp\left[-(2k+1)^2q^2/8(1+S)\right], & k \neq 0 \end{cases} \tag{15}$$

$$F_0(c_k) = \begin{cases} 0, & k = 0 \\ 1 - \exp\left\{-(2k-1)^2q^2/8\right\}, & k \neq 0 \end{cases} \tag{16}$$

We will select N = 30, and we will use a threshold T = 341 corresponding to a $P_{FA} \simeq 1.097 \; 10^{-6} (\sim 10^{-6})$. This threshold has be found, by numerical convolution, the value that offers the P_{FA} nearest to 10^{-6} without randomizing in thresholding. The following table summarizes corresponding results: the first row shows $P_{DT}(S)$ ("exact" value) calculated by numerical convolutions of {1}

$$\Pr\left[R_{i+i',j} = 1 \mid S\right] = \binom{M}{1} \sum_{i=0}^{1} (-1)^i \frac{1}{(1+S)(M-1+i)+1} \tag{17}$$

the rank distribution in the continuous case. The second row shows $P_D(S)$ under Gaussian assumption, using m(S) and {11}

$$\sigma^2(S) = M^2\left\{\int_0^\infty F_0^2(X)\,dF_s(X) - \left[\int_0^\infty F_0(X)\,dF_s(X)\right]^2\right\} + M\left[\int_0^\infty F_0(X)\,dF_s(X) - \int_0^\infty F_0(X)\,dF_s(X)\right] \tag{18}$$

For a Swerling II target, we have {11}

$$\int_0^\infty F_0(X)\,dF_s(X) = (1+S)/(2+S) \tag{19a}$$

by restricting RMs; after this, we can select among them and determine q in accordance with the effect of $\tilde{\sigma}(0)$ ($\tilde{P}_{FA}$) and the values of $\tilde{P}_D(S)$.

III. SOME RANDOMIZATION METHODS AND VARIANCE CALCULATIONS

Three RMs verifying (9) are the following:

A) $A_{k,c}(l)$ dichotomic: 0 with probability 1/2, c with probability 1/2;

B) $A_{k,c}(l)$ binomial: l with probability $\binom{c}{l}/2^c$ $(l = 0,\ldots,c)$;

C) $A_{k,c}(l)$ uniform between 0 and c.

Condition (9) will be satisfied if Q is great enough to do not consider the limiting effect on the expression giving $\tilde{m}(0)$; i.e., since $\mu(k, c) = c/2$ and, then, $\bar{\bar{\mu}}(k) = p_k(0)/2$,

$$\tilde{m}(0) = M \sum_{k=0}^{Q-1} p_k(0)\left[F_0(c_k) + p_k(0)/2\right] = M \sum_{k=0}^{Q-1} \left[F_0(c_{k+1}) - F_0(c_k)\right]\left[F_0(c_{k+1}) + F_0(c_k)\right]/2 \tag{10}$$

and (10) must approximate (9), the sum from 0 to Q - 1 approximates $\int_0^\infty F_0(X)\, d\, F_0(X)$ by the mean value of F_0 in each interval (c_k, c_{k+1}) multiplied by the interval amplitude; since the sub-integral function is linear in F_0, this is an "exact" representation when Q is great enough ($F_0(c_{Q-1}) \approx 1$). We will assume this in the following.

It is possible to use RMs in which an 1 is taken with probability P_k (depending on the quantizing level) in the place of 1/2 in cases A and B; but the above discussion shows that the best selection is $P_k = 1/2$ for all k.

Simple but tedious calculations lead to the following expressions for the variance of $R_{i+i',j}$ (subindexes indicate RM)

$$\tilde{\sigma}_A^2(S) = M^2\left[\beta(S) + \gamma(S)/4 - \alpha^2(S)\right] + M\left[\alpha(S) - \beta(S) - \gamma(S)/4\right] \tag{11a}$$

$$\tilde{\sigma}_B^2(S) = M^2\left[\beta(S) - \alpha^2(S)\right] + M\left[\alpha(S) - \beta(S)\right] \tag{11b}$$

$$\tilde{\sigma}_C^2(S) = M^2\left[\beta(S) + \gamma(S)/12 - \alpha^2(S)\right] + M\left[\alpha(S) - \beta(S) - \gamma(S)/12\right] \tag{11c}$$

where

$$\alpha(S) = \sum_{k=0}^{Q-1} p_k(S)\left[F_0(c_k) + p_k(0)/2\right] \tag{12a}$$

$$\beta(S) = \sum_{k=0}^{Q-1} p_k(S)\left[F_0(c_k) + p_k(0)/2\right]^2 \tag{12b}$$

$$\gamma(S) = \sum_{k=0}^{Q-1} p_k(S)\, p_k^2(0) \tag{12c}$$

(5) and (11a, b, c) can be calculated by means of a computer program by consecutive additions; we can stop the computation when k = K obtaining a

$$\int_0^{\infty} F_0^2(X)\, dF_s(X) = 1/(3 + 2S) \tag{19b}$$

The remaining rows correspond to randomizations A, B, C and q = 2, 1, 3/4, 1/2, 1/3 and 1/10.

On these results, we can remark the following main observations for the example:

* There are not important differences among the three RMs in the region of interest for $\tilde{P}_D(S)$; method B shows the lowest $\tilde{P}_{FA}$ and method C the less variable P_{FA}.
* A maximum quantizing step $q \simeq 1/2$ is necessary to adequately maintain the approximate continuous values (signal loss about 0.2 dB, $\tilde{P}_{FA}$ within the order of magnitude of P_{FA}).

In general, it seems that RMs will not be important if they are selected verifying the mean condition $m(0) = \tilde{m}(0)$ for acceptable values of q, and that $q \lesssim 1/2$ (approximately) is needed to avoid parametric effects. Of course, these results must be checked under other conditions.

CONCLUSION AND FURTHER WORK

We have introduced a formulation that allow, by means of some approximations, to evaluate and select quantizing steps and RMs to be used in a GST Radar detector easily. The work is being extended to consider

* other kinds of targets;
* margins of validity of Gaussian assumption in function of M and N;
* asymptotic behaviours;
* other RMs;
* other NP Radar detectors.

Futher extesions will try to include some approximations to consider the effects of MTI and digital implementations of the envelope detectors, and the determination of the dynamic range by applying a similar formulation to situations in which the problem is saturation (upper limitation), with certain clutter models.

	S(dB)	-2	-1	0	1	2	3	4	5	6	P_{FA}, $\tilde{P}_{FA}$
$P_{DT}(S)$		1.54	1.44	4.75	0.135	0.262	0.458	0.668	0.838	0.937	1.097
P_D (S)		0.843	2.22	5.55	0.126	0.254	0.441	0.653	0.833	0.941	2.18
$\tilde{P}_D(S)$ q= 2	A	0.406	0.777	1.54	0.031	0.063	0.123	0.226	0.377	0.563	67.6
	B	0.012	0.040	0.142	0.005	0.018	0.058	0.157	0.339	0.582	0.095
	C	0.066	0.160	0.474	0.012	0.033	0.083	0.185	0.355	0.574	2.18
q= 1		0.950	2.07	4.48	0.094	0.183	0.324	0.509	0.703	0.859	33.0
		0.253	0.777	2.30	0.062	0.148	0.300	0.510	0.727	0.887	0.267
		0.437	1.16	2.87	0.071	0.161	0.309	0.510	0.718	0.877	2.18
q=3/4		0.909	2.13	4.89	0.106	0.209	0.369	0.567	0.760	0.898	12.5
		0.429	1.25	3.41	0.086	0.189	0.358	0.573	0.776	0.914	0.702
		0.558	1.52	3.90	0.093	0.196	0.362	0.571	0.770	0.909	2.18
q=1/2		0.879	2.18	5.20	0.116	0.232	0.406	0.613	0.800	0.924	5.27
		0.630	1.72	4.47	0.106	0.223	0.403	0.617	0.809	0.930	1.32
		0.709	1.87	4.72	0.110	0.226	0.404	0.616	0.806	0.928	2.18
q=1/3		0.861	2.20	5.39	0.121	0.244	0.425	0.635	0.819	0.934	3.32
		0.742	1.98	5.03	0.117	0.240	0.424	0.637	0.822	0.937	1.74
		0.781	2.05	5.15	0.118	0.241	0.424	0.637	0.821	0.936	2.18
q=1/10		0.845	2.22	5.51	0.126	0.253	0.439	0.652	0.832	0.941	2.27
		0.834	2.19	5.48	0.125	0.252	0.439	0.652	0.832	0.942	2.13
		0.838	2.20	5.49	0.125	0.253	0.439	0.652	0.832	0.941	2.18
		$(\times 10^{-2})$	$(\times 10^{-2})$	$(\times 10^{-2})$							$(\times 10^{-6})$

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