PERFORMANCE OF A FH MULTILEVEL FSK FOR MOBILE RADIO IN THE PRESENCE OF NONSYNCHRONOUS USERS

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ABSTRACT

This paper considers a Frequency Hopped Multilevel Frequency Shif Keying (FH-MFSK) spread spectrum communication system applied to mobile radio-telephony. We present a mobile-to-base transmission model that allows to study system impairments, such as interference from nonsynchronous users and adjacent frequency channels in presence of matched tuned receiver filters.

We have obtained results with mobile-to-base communication of 32 Kb/s per user in a 20 MHz (one-way) bandwidth. A bit error probability less than 10^{-3} can be maintained with up to 110 simultaneous users for practical average SNR ratio of 25 dB.

INTRODUCTION

In order to provide digital mobile radiotelephony communication services to a great number of users, spread spectrum modulation techniques using Frequency Hopped (FH) Frequency Shif Keying (FSK) have been investigated recently (ref.1).

Assigning to each user a different tone sequence as his address, the FH-MFSK system allows many users to share the same frequency band. Because the address is spread in frequency, the system behaves well in the presence of selective fading, making it particularly suitable for mobile service in an urban environment. Nevertheless, its performance is limited by mutual interference between users.

In this paper we introduce a new model to study this interference. We analize the mobile-to-base transmission by nonsynchronous users, tuned matched filtering at the receivers and interchannel interference.

Throughout our discussion, we assume the existence of a power control mechanism (ref.2) that makes it possible for the base station to receive the same average power from each mobile assigned to it. This power control althrough not strictly necessary for the system operation, it would nevertheless enhance the FH-MFSK system performance by easing design constraints on the RF preamplifier and by limiting intercell interference in a multicel system. It would also serve to assign to each mobile the most appropriate base station for adequate reception.

The results obtained are, in general, in agreement with those in (ref.1), and extend them in some directions.

SYSTEM DESCRIPTION

In this multiple-access modulation scheme, every T seconds, each user transmits his information in blocks of K bits. For this purpose the system has available 2^K different frequencies numbered $0,1,\ldots,2^K-1$. With no other users, message transmission requires only one time interval of T seconds duration, and it is accomplished by the obvious assignment of messages to frequencies. With M>1 simultaneous users, L>1 intervals of duration $\tau=T/L$ are used, and frequency hopping is employed to allow communication in the presence of interference from other users. During the basic signaling interval T, the mth. user (the subscript m denotes one link in a multi-user system) has an address generator that generates a sequence of L numbers, each K bits long: $V_{m,1},V_{m,2},\ldots,V_{m,L}$.

Each user m is assigned an unique sequence $V_{m,q}(q=1,\ldots,L)$ which is used to distinguish his messages from those of others. We also refer to this sequence as the address vector of user m. The transmitted tone sequence, at the rate of one tone (chip) every τ seconds, is assigned by the modulo 2^K sum ($\oplus$) of the address and the K-bit code word X_m.

$$Y_{m,q}= X_m \oplus V_{m,q} \qquad q=1,2,\ldots,L \qquad (1)$$

At the receiver, demodulation and modulo 2^K subtraction (θ) by $V_{m,q}$ are performed every τ seconds, yielding

$$Z_{m,q}= Y_{m,q} \;\theta\; V_{m,q}= X_m \qquad (2)$$

The sequence of operations is illustrated by the matrices of Fig.1 and 2 (see also ref.1), where the 2^K tones have been placed at intervals of $1/\tau$ Hz.

Noise, multiuser interference, interchannel interference, etc., can influence the detection matrix by causing a tone to be detected when none has been transmitted

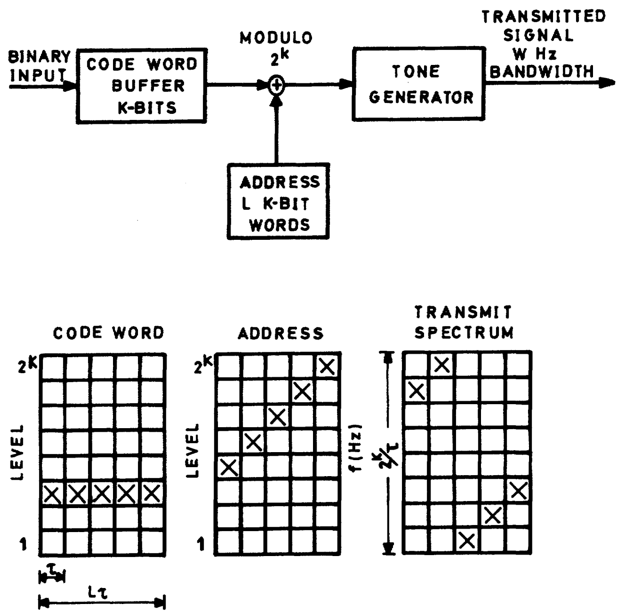


Fig.1. Transmitter FH-MFSK.

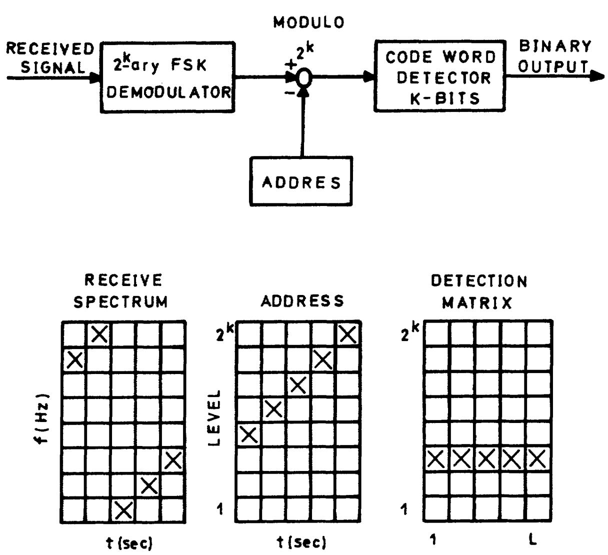


Fig.2. Receiver FH-MFSK.

(insertion). In addition, the receiver can omit a transmitted tone (miss) and cause a detection matrix to have no complete row. To allow for this possibility, we use the majority logic decision rule: Choose the code word associated with the row containing the greatest number of entries.

The computation of bit error probability, P_B, can be carried out knowing the insertion probability, P_I, and the miss probability, P_{MISS}, as we show in the Appendix.

TRANSMISSION MODEL

If we assume that the address code for each mobile is randomly generated, the probability, conditioned on the existence of only one user, that a frequency corresponding to the frequency channel j ($1 \leqslant j \leqslant 2^K$) is present in a given interval of duration τ is: $P=2^{-K}$.

Consider now the presence of M nonsynchronous users transmitting simultaneously. In any interval of duration τ there are M randomly generated epochs, or time instants, coinciding with the changes in the users chip frequency transmission. The probability that in the given interval there are N epochs in which the new chip frequency transmission corresponds to the frequency channel j is

$$P_N = \binom{M}{N} p^N (1-p)^{M-N} \qquad (3)$$

If $M \gg 1$ and $p \ll 1$ we can approximate the Binomial distribution by a Poisson distribution (ref.3) giving

$$P_N \simeq \frac{(Mp)^N}{N!} \exp(-Mp) \qquad (4)$$

Hence, the average number of epochs in the frequency channel j during an interval of duration τ is

$$Mp = \lambda\tau \qquad (5)$$

That is, we can model the multiuser interference as an uniform Poisson process of parameter

$$\lambda = \frac{M}{2^K \tau} \qquad (6)$$

The above approach allows us to write the multiuser interference signal present at the receiver input in any chip interval of the desired signal as

$$z(t) = \sum_{i=-\infty}^{\infty} \sum_{j=1}^{2^K} R_{i,j} \cos\left[2\pi f_j (t-t_{i,j})+\theta_{i,j}\right] \cdot \mathrm{rect}_\tau(t-q\tau-t_{i,j}) \qquad (7)$$

$$q\tau \leqslant t < (q-1)\tau, \quad q \text{ integer}$$

where the f_j is the frequency corresponding to the frequency channel j.

The $\{R_{i,j}\}$ are statistically independent random variables and each is Rayleigh distributed (short-term fading). The $\{\theta_{i,j}\}$

are statistically independent random variables and each is uniformly distributed over $[0,2\pi]$. Moreover, there are independent of the $\{R_{i,j}\}$.

$$\mathrm{rec}_\tau(t) = \begin{cases} 1 & 0\leq t<\tau \\ 0 & \text{otherwise} \end{cases}$$

$\{t_{i,j}\}$ is a Poisson random point process corresponding to frequency channel j. The sequence of epochs, which is due to nonsynchronous users, distinguishes this model from the base-mobile model used in (ref.1) where the arrival times are synchronized to the beginning of the chip.

Power control is employed which maintains the mean power received by each station from mobiles to $P=E(R_{i,j}^2)/2$. This power control eliminates the changes in signal level due to path loss variations and the effects of shadow fading.

We have also assumed that all the arrival tones in the frequency channel j fade independently of each other, because fading tends to be caused by phenomena in the vicinity of the mobile.

The received complex envelope signal at the base station in the, say, frequency channel j (the subindex j will be omitted from now on for the sake of clarity in the notation) is:

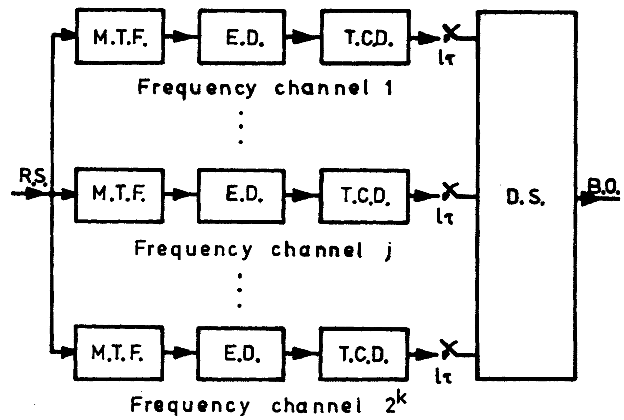
$$z_I(t) = R\,\exp(-j\theta)\,\mathrm{rect}_\tau(t) + \sum_{i=-\infty}^{\infty} R_i \exp(-j\theta_i)\,\mathrm{rect}_\tau(t-t_i)+n(t) \quad (8)$$

$0\leq t<\tau$

The first term in the above formula corresponds to the desired signal. The second, to the multiuser interference signal and n(t) is the white gaussian noise present at the 2^K-ary FSK receiver input. This receiver sets up a tuned filter on the 2^K transmitted frequencies (see Fig. 3). Every filter is followed by an envelope detector and a threshold decision circuit.

The presence of nonsynchronous users prevents the adoption of modulation schemes with 2^K orthogonal tones, each of duration τ seconds, maintaining a bandwidth of at least $W=2^K/\tau$ Hz.

If the frequency channel spacing adopted is the reciprocal of the chip interval, then an additional impairment appears due to adjacent channel interference (a bandwidth penalty would be needed to remove it). If we denote by h(t) the complex envelope impulse response of the filter corresponding to the desired frequency channel, the complex envelope signal at the output, taking into account the interference due to the IR higher adjacent channels and the IL lower channels is



R.S. = Received Signal
M.T.F. = Matched Tuned Filter
E.D. = Envelope Detector
T.C.D. = Threshold Circuit Decision
D.S. = Decision System
B.O. = Binary Output.

Fig.3. 2^K-ary FSK Demodulator.

$$z_0(t)=\underbrace{R\,\exp(-j\theta)\,S_0(t)}_{1}+r(t)+n_f(t) \quad 0\leq t<\tau \quad (9)$$

with

$$r(t)=\underbrace{\sum_{i=-\infty}^{\infty} R_i \exp(-j\theta_i)\,S_0(t-t_i)}_{2}$$

$$\underbrace{+\sum_{\substack{c=-IL\\ c\neq 0}}^{IR}\ \sum_{v=-\infty}^{\infty} R_{v,c}\exp(-j\theta_{v,c})\,S_c(t-t_{v,c})}_{3} \quad (10)$$

where 1) is the desired signal, 2) is the multiuser interference signal, and 3) is the adjacent channel multiuser interference signal.

$$S_c(t)=\frac{1}{2}\,\mathrm{rect}_\tau(t)\,\exp(-j2\pi\frac{c}{\tau}t) * h(t) \quad (11)$$

$c=-IL,\ldots,0,\ldots,IR$

The $\{t_{v,c}\}$ is a Poisson random point process corresponding to c adjacent frequency channel.

The $\{R_{v,c}\}$ and $\{\theta_{v,c}\}$ are identically distributed random variables as the $\{R_{i,j}\}$

and $\{\theta_{i,j}\}$ previously mentioned.

$$n_f(t) = \frac{1}{2}\, n(t)*h(t) = n_x(t) + j\, n_y(t) \qquad (12)$$

$Z_0(t)$ is sampled every τ seconds in order to fill the decoding matrix and perform the majority logic decision rule every T.

CALCULATION OF THE MISS AND INSERTION PROBABILITIES

As we already stated, the calculation of the bit error probability requires the previous computation of the insertion and miss probabilities, which we now outline.

Refering to Fig.3 and considering the filtered signal, r(t), mentioned as eq. 10, we have that in absence of the desired signal, the insertion probability is

$$P_I = \Pr(|r(\tau)+n_f(\tau)| \geq C_0) \qquad (13)$$

where C_0 is the threshold decision value.

Defining $r=|r(\tau)|^2$, we can write (ref.4)

$$P_I = \int_0^\infty Q\left(\frac{\sqrt{x}}{\sigma_n}, \beta\right) f_r(x)\, dx \qquad (14)$$

where
Q(.,.) is the Marcum Q function

$$\sigma_n^2 = \frac{1}{2} E\left[|n_f(\tau)|^2\right] \qquad (15)$$

$$\beta = \frac{C_0}{\sigma_n} \qquad (16)$$

$f_r(.)$ is the probability density function of the random variable r.

We have performed the calculation to determine P_I using a Gaussian Quadrature Rule (GQR), which guarantees, in the area of approximate integration, the highest degree of precision.

$$P_I \simeq \sum_{u=1}^{N} Q\left(\frac{\sqrt{x_u}}{\sigma_n}, \beta\right) w_u \qquad (17)$$

x_u are called the abscissas of the formula and the w_u the weights, so the set $\{w_u, x_u\}_{u=1}^N$ is called a quadrature rule corresponding to the weight function $f_r(x)$.

The lack of knowledge on $f_r(x)$, as in our case, can be bypassed using the algorithm introduced by Golub and Welsch (ref. 5) that performs the computation of $\{w_u, x_u\}_{u=1}^N$ using the 2N+1 moments of the random variable r:

$$m_n = E(r^n) \qquad n=0,\ldots,2N \qquad (18)$$

Analogously, as we did with P_I, we can write the miss probability as

$$P_{MISS} = \Pr(|R \exp(-j\theta)\, S_0(\tau) + r(\tau)+n_f(\tau)| < C_0) \qquad (19)$$

Given that $R\, S_0(\tau)\cos\theta$ and $R\, S_0(\tau)\sin\theta$ are independent Gaussian random variables, we can include them in $n_x(\tau)$ and $n_y(\tau)$ respectively and define the new variance as:

$$\sigma_T^2 = \sigma_n^2 + \frac{E(R^2)}{2}\, |S_0(\tau)|^2 \qquad (20)$$

Then, we have

$$P_{MISS} = 1 - \int_0^\infty Q\left(\frac{\sqrt{x}}{\sigma_T}, \beta\right) f_r(x)\, dx \qquad (21)$$

The computation of the above formula can be carried out by the GQR already used to calculate P_I.

NUMERICAL RESULTS

This section contains some numerical results and curves for the design variables: K=8, L=19 and W=20 MHz, which allow a transmission rate per user of $K/L\tau \simeq$ 32Kb/s and were chosen in (ref.1) as optimum.

Since with GQR the convergence to the true value of the bit error probability is guaranteed, it is sufficient to check how many significant digits remain unchanged as N increases, and continue the iteration until the desired accuracy. In the numerical results of this section the convergence was obtained, at least, up to the first two significant digits.

P_B depends, among other factors, on β. Let βopt be the value of β that minimizes P_B.

βopt depends strongly on the number of users and the average chip energy-to-one sided power spectral noise density ratio Ec/No. For the βopt (see table 1) calculated in the presence of the adjacent frequency channels (IL=IR=4) we have computed P_B and plotted it in Fig.4.

TABLE 1

βopt with matched receiver filters and adjacent interchannel interference. (IL=IR=4).

M \ Ec/No	5	10	15	20	25	30
50	2.5	3	3.5	5	7.5	12
75	2.75	3	3.75	5.5	8.5	13.5
100	2.75	3	4	6	9.25	15
125	2.75	3.25	4.25	6.5	10.25	16.75
150	2.75	3.25	4.25	6.75	10.75	17.5
175	3	3.25	4.5	7	11.25	18.75
200	3	3.25	4.75	7.5	12	19.75

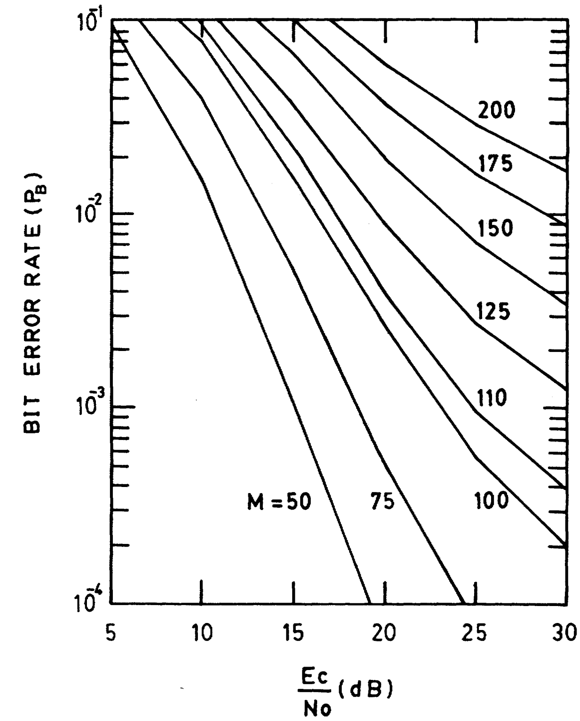


Fig.4. Bit error probability with adjacent interchannel interference (L=19).

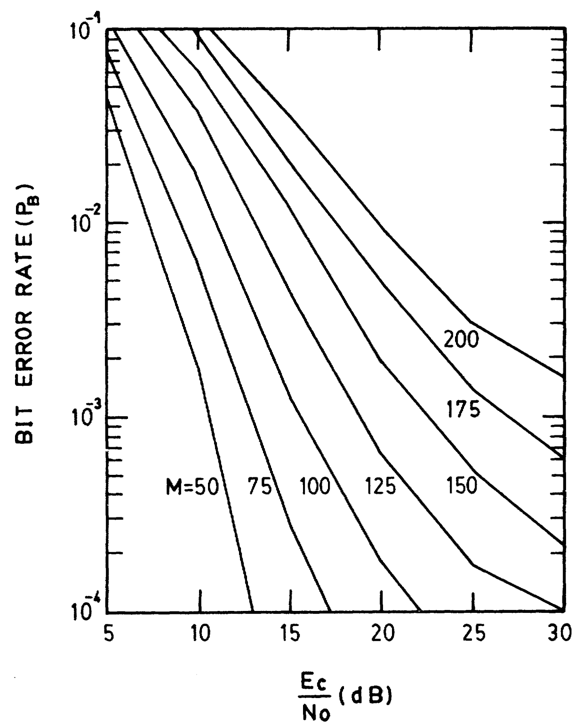


Fig.5. Bit error probability assuming no adjacent interchannel interference (L=19).

Fig.5 shows the P_B values obtained assuming no interchannel interference. These results agree very well with those given by the synchronous users model studied in (ref.1).

Comparing Fig.4 and 5, we observe that the presence of interchannel interference in our nonsynchronous model introduces an important degradation. For instance, with typical SNR=Ec/No=25 dB and for $P_B<10^{-3}$, the system can accomodate up to $M \simeq 170$ users without adjacent interchannel interference and up to 110 users with it.

We have also considered L values different of L=19 maintaining the design variables W=20 MHz, K=8 and the transmission rate of 32 Kb/s. Since $L\tau$ is fixed,

$$a = \frac{19}{L} \qquad (22)$$

Then, the interchannel spacing is given by a/τ.

In Fig.6 P_B is plotted for different M and L values. A typical SNR=25 dB has been considered. If M>125, the optimum L value is 19 (a=1). Lower interchannel separations are optimum for M<125. In these cases the lower user density counteract the greater interchannel interference and the optimum performance is reached with L>19.

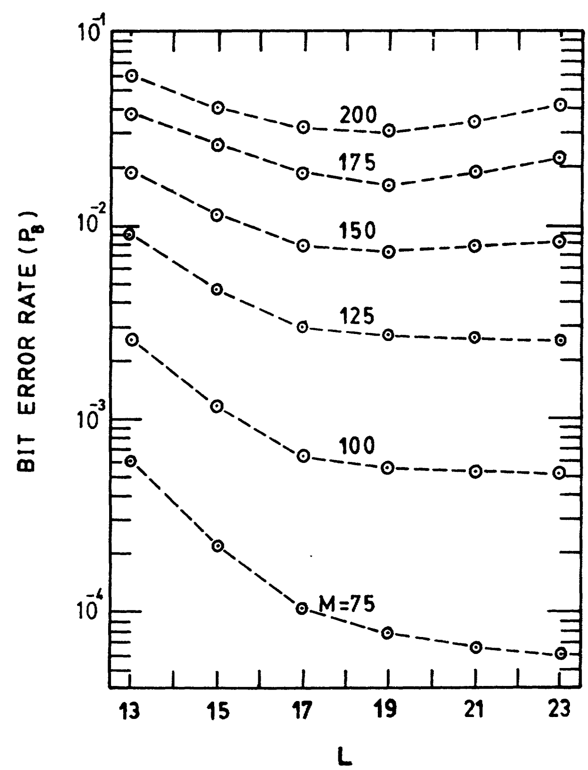


Fig.6. Bit error probability for several interchannel separations (SNR= 25 dB).

CONCLUDING SUMARY

In this paper we have presented a transmission model for a FH-MFSK digital land mobile radiotelephony system that allows to incorporate a great number of impairments. In particular, a novel interference model has been introduced that permits to analyse mobile-to-base communications, taking into account the interference from nonsynchronous users and adjacent channels with matched receiver filters.

Results were obtained by integrating the condicional error probability with respect to the density function of interference using the Gaussian Quadrature Rule method.

The performance of the FH-MFSK multiple access system for mobile radio has been plotted on a set of curves which display the bit error rate (P_B) versus the signal to noise ratio (SNR) using the number of simultaneous users as parameter (M). For typical SNR=25 dB, a 20 MHz bandwidth and 32kb/s per user, different chip interval durations have been also considered and insight in its effects on P_B were gained.

APPENDIX

Calculation of the bit error probability.

Under the majority logic decision rule, an error occurs when insertions combine to form unwanted rows with more entries than the correct row of the detection matrix of Fig.2. An error can occur when insertions combine to form unwanted rows with the same number of entries as the correct row. Where there are j unwanted rows with the same number of inputs that the correct row, a row is chosen at random. Then, if we assume that all the rows have the same probability of being selected, the probability of having the correct one is:1/1+j.

Assuming that the word error probability, P_w, is known, the bit error probability is given by

$$P_B = \frac{2^{k-1}}{2^k-1} P_W \qquad (23)$$

Now we show how to obtain P_W from P_I and P_{MISS} assuming that all the frequency channels fade independently (selective fading). Since $P_w=1-P_{NE}$, we have

$$P_{NE} = \sum_{q=0}^{L} P(NE/q)\ P_C(q) \qquad (24)$$

$$P_C(q) = \binom{L}{q} P_{MISS}^{(L-q)} (1-P_{MISS})^q \qquad (25)$$

where P(NE/q) is the probability that a word is error free conditioned on the presence of q entries in the correct row.

$P_C(q)$ is the probability of having q entries in the correct row.

Under the majority logic decision rule:

$$P(NE/q) = \sum_{j=0}^{2^k-1} \left(\frac{1}{1+j}\right) P(q,j) \qquad (26)$$

where P(q,j) is the probability that j non wanted row have q entries and the other 2^k-j-1 rows have less than q entries.

Defining $P_S(q)$ and $Q_S(q)$ as the probabilities of having q or fewer entries respectively in a spurious row. We have

$$P(q,j) = \binom{2^k-1}{j} P_S^j(q) \left[Q_S(q)\right]^{2^k-j-1} \qquad q\neq 0 \quad (27)$$

$$P(q,j) = \left[(1-P_I)^L\right]^{2^k-1} \qquad q=0, j=2^k-1 \quad (28)$$

$$P(q,j) = 0 \qquad q=0, j\neq 2^k-1 \quad (29)$$

Finally

$$P_S(q) = \binom{L}{q} P_I^q (1-P_I)^{L-q} \qquad (30)$$

$$Q_S(q) = \sum_{n=0}^{q-1} P_S(n) \qquad q\neq 0 \quad (31)$$

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