# Applicability of the Lorentzian peak method to analyze leaky and lossy optical waveguides

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*Several conclusions concerning the applicability of the Lorentzian peak method to compute the complex propagation constant of leaky and lossy waveguide modes are established.*

Usually the calculation of the loss coefficient of leaky and lossy modes appearing in common integrated optical technology constitutes a very cumbersome and long process. This difficulty is a result of such modes being complex guided ones, so that their propagation constant exhibits both real and imaginary parts. To find the effective index of these modes ($N = N_r + jN_i$), complicated transcendental equations have to be solved in most cases. This point requires a high number of iterations in the complex plane, and as a consequence a great volume of calculations are generated. Recently, an approximate procedure, which is referred to as the Lorentzian peak method (LPM), has been reported[1] to analyze this problem. This method is based on the fact that for the cases of practical interest the imaginary part of the effective index amounts to very small values. Typically $N_r \sim 1$ and $N_i \sim 10^{-4,-5}$. Unfortunately, there is no clear criterion to establish the applicability of the method to a particular structure. The main aim of this work is to address this question by means of a slightly different approach than the original procedure.

The eigenvalue equation for a general waveguiding system can be written as $D(N) = 0$, $D(N)$ being a transcendental complex function. Accordingly, the effective indices of the modes supported by the waveguiding structure are obtained as the real and complex zeros of that function. So, if $N_m = N_{rm} + jN_{im}$ is the effective index of a given mode, one has $D(N_m) = 0$, and in the neighborhood of $N_m$ the function $D(N)$ can be approximated as $D(N) \simeq A(N - N_m)$, $A$ being a complex constant. Then, introducing the function $f(N_r) \equiv |D(N_r,N_i = 0)|^2$, which is only defined along the real $N$-axis, one obtains $f(N_r) \simeq |A|^2[(N_r - N_{rm})^2 + N_{im}^2]$.

In a first approach (LPM), the function $1/f(N_r)$ can be plotted to obtain Lorentzian behavior. Using the fact that $N_{im} \ll 1$, it can be assumed that approximate values of $N_{rm}$ and $N_{im}$ can be obtained by curve-fitting the resulting plot.[1] However, usually the values found for $N_{rm}$ and $N_{im}$ using this procedure depend on the sample range used in the curve-fitting process. This difficulty can be avoided by taking into account that $f(N_r)$ can be written also as

$$f(N_r) \simeq f(N_{rm})\left[1 + \left(\frac{N_r - N_{rm}}{N_{im}}\right)^2\right]. \qquad (1)$$

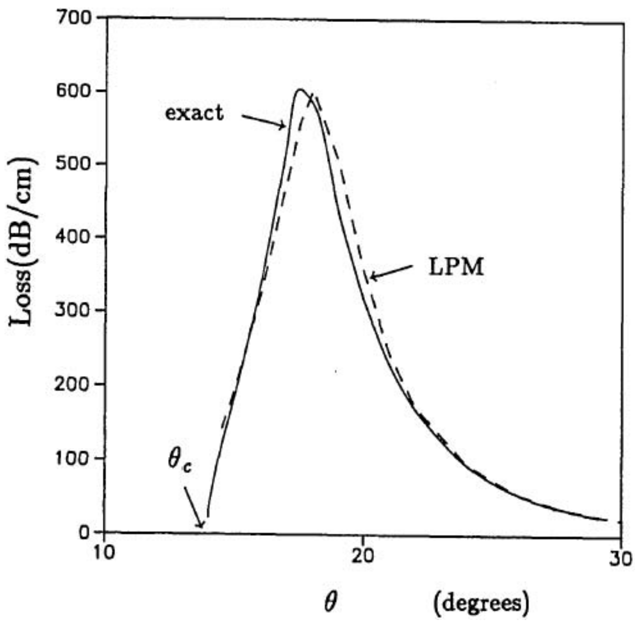


Fig. 1. Loss coefficient as a function of the optical axis orientation for the step index waveguide ($t = 2\ \mu$m). We have not included the strongly inaccurate LPM results at $\theta \simeq \theta_c$.

Now, the $N_{rm}$ and $f(N_{rm})$ are known, this equation defines a uniparametric real function, and $N_{im}$ can be straightforwardly calculated by a simple linear regression procedure. Also, it allows us to obtain approximate values of $N_{rm}$ and $f(N_{rm})$, since $f(N_r)$ exhibits a relative minimum at $N_r = N_{rm}$. This is a single and objective value, i.e., it does not depend on the used sample range, and thus reproducible results are obtained. The amount of calculation required is similar to what is needed in the usual root-finding schemes for the pure guided modes ($N_{im} = 0$), since the iteration procedure can be performed by a fast converging minimizing algorithm.

To check the applicability of the above procedure we have considered the off-axis propagation in a typical $X,Y$-cut $LiNbO_3$-based waveguide, in which the crystal optical axis lies in the waveguide plane making an angle $\theta$ with the waveguide axis. The corresponding eigenvalue equation has been derived from the standard transfer-matrix formalism.[2] First, we consider a step index waveguide of thickness $t$ in which the cover is in air. The waveguide parameters are $n_{of} = 2.2946$, $n_{ef} = 2.2108$, $n_{os} = 2.2866$, $n_{es} = 2.2028$, $t = 2\ \mu$m, and $\lambda = 633$ nm. Here $n_{os}$,$n_{es}$ are, respectively, the ordinary and the extraordinary refractive index of the substrate, whereas $n_{of}$,$n_{ef}$ correspond to the film. When $\theta = 0°$, this waveguide supports the $TE_0$ and $TM_0$ modes. The $TE_0$ mode at $\theta = 0°$ remains guided for all values of $\theta$; meanwhile the mode which is the $TM_0$ at $\theta = 0°$ becomes leaky beyond $\theta_c \simeq 14°$. In Fig. 1 we plotted the loss coefficient of this mode as a function of $\theta$. The continuous line corresponds to the exact solution obtained by directly solving the eigenvalue equation by means of a numerical zoom root-finding algorithm. The dashed line follows from the modified LPM.

Table I. Inhomogeneous Waveguide

| $\theta$ | $N_{rm}$ | | Loss (dB/cm) | |
|---|---|---|---|---|
| | exact | LPM | exact | LPM |
| 80° | 2.20805 | 2.20805 | 0.9 | 0.9 |
| 60° | 2.22573 | 2.22573 | 8.9 | 9.1 |
| 30° | 2.26757 | 2.2647 | 54.0 | - |
| 20° | 2.27913 | 2.2763 | 110.9 | - |

The agreement between both sets of values is excellent, even in the region of highest losses.

Also, the error in the calculation of $N_{rm}$ amounts to a negligible value in most cases, so that it is only noticeable in the region where $N_{im}$ has great values. This is because, generally speaking, for the points belonging to the real $N$-axis, the linear approximation for $D(N)$ becomes less accurate when the actual root moves away from this real axis. The error is reduced to a negligible value by making a second calculation by means of the new function $f_1(N_r) \equiv |D(N_r,N_{im}^0)|^2$, $N_{im}^0$ being the value obtained at the first step. Also, it is worth noticing that the LPM does not work in the region $\theta \sim \theta_c$, since this is a cutoff point and the actual roots stem from a sharp structure of $D(N)$ in the complex plane. On the other hand, very good agreement has also been observed for a multimode version ($t$ = 3 $\mu$m) of the above example. This fact has to be emphasized, since in this case the loss coefficient shows a complicated behavior as a function of $\theta$ with various maxima and minima.[3]

Now, we are going to examine an inhomogeneous waveguide with a Gaussian profile in both $n_{of}$ and $n_{ef}$, which supports also a pure guided mode and a leaky guided mode. The waveguide parameters are identical as in the former single-mode example. Again, the accuracy obtained in this case using the modified LPM is very good when $N_{im}$ for the leaky mode has small values. Nevertheless, the results in the regions with moderately high losses strongly disagree with the exact values.[2] Table I shows some calculated values. This important disagreement occurs in a range of ~25° beyond $\theta_c \simeq 11°$ and comes from the particular form of the function $D(N)$. In Fig. 2 the shape of the function $f(N_r)$ in the neighborhood of $N_{rm}$ is shown for $\theta$ = 80°. The plot has two minima. The first one, which occurs for the smaller value of $N_r$, is a local minimum and does not correspond to any root of the equation $D(N) = 0$. The remaining minimum, which occurs at a higher value of $N_r$ (on the right of the steep maximum), is the one associated with the actual zero of the function $D(N)$ and leads to the results given in Table I.

The shape of function $f(N_r)$ shown in Fig. 2 is the same, whatever the value of $\theta$. Nevertheless in the range $\theta_c < \theta \leq 37°$, the zero of $D(N)$ produces a very sharp minimum in the complex plane to such an extent that, although it occurs for typical values of $N_{im} \sim 10^{-4}$, its existence is not revealed in the axis $N_i = 0$. In these conditions, in the plot equivalent to the one shown in Fig. 2 for $f(N_r)$, the correct minimum does not appear, leading to the failure of the basic assumption of

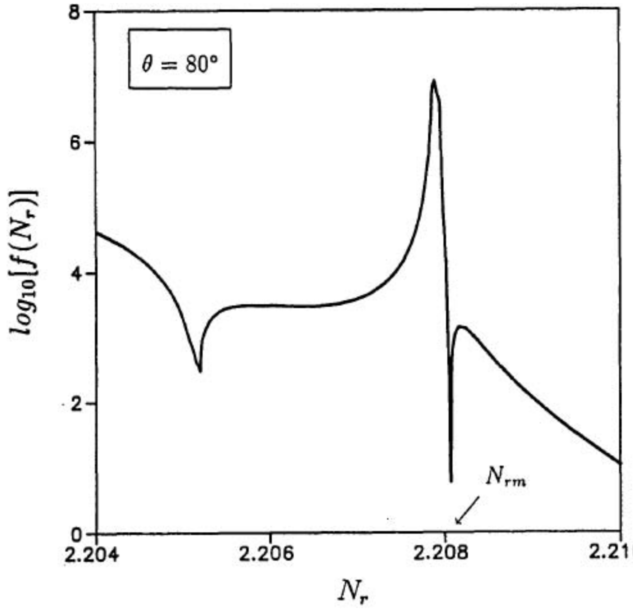


Fig. 2. Decimal logarithm of $f$ as a function of $N_r$ for the inhomogeneous waveguide with $\theta$ = 80°. $N_{rm}$ = 2.20805 corresponds to the actual root of the function $D(N)$ for the leaky mode; for the pure guided mode $N_{rm} = N_m$ = 2.28942.

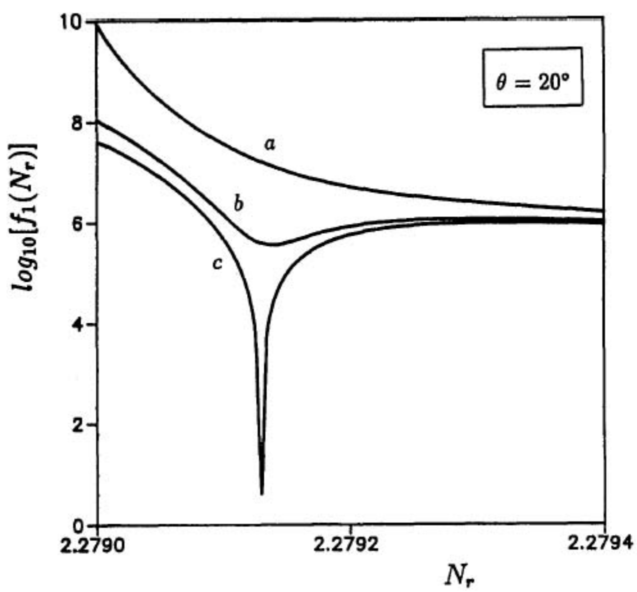


Fig. 3. Decimal logarithm of $f_1$ as a function of $N_r$ for the inhomogeneous waveguide with $\theta$ = 20°: $a, N_{im}^0 = 0$; $b, N_{im}^0 = 10^{-4}$; $c, N_{im}^0 = 1.3 \times 10^{-4}$.

the LPM. This behavior can be clearly seen in Fig. 3 for $\theta$ = 20°. However, in the above mentioned harmful range of $\theta$ values, the function $f(N_r)$ still shows one local minimum similar to the one appearing in Fig. 2, which, as already pointed out, does not correspond to any zero of $D(N)$. This local minimum misleads the minimization algorithm and leads to the erroneous values of $N_{rm}$ shown in Table I. The obtained values of $N_{im}$ in these conditions make no sense, so they are not included in the table.

In conclusion, our results show that the LPM provides very accurate results in various cases with small computer times. Unfortunately, it does not work in other cases, when the eigenvalue equation shows a sharply peaked or oscillatory behavior in the complex plane. These results have deep implications because they show that when the LPM is applied to a given leaky or lossy structure, just to obtain a good agreement in a partial check of the approximate results is not enough to guarantee the viability of the procedure when the method is applied to a different system. In fact, it is not obvious how to *a priori* establish the conditions at which the function $D(N)$ exhibits the harmful behavior leading to the failure of the procedure. However, it seems that the problematic cases follow from special situations to be identified, a question that can only be answered after further investigation. Finally, we have shown also that when this harmful behavior does not occur, the LPM constitutes a very powerful computational tool.

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