Corrigendum to “Particle production from marginally trapped surfaces of general spacetimes” (Class. Quantum Grav. **32** (2015) 085004)

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Abstract

We correct some computations presented in section 4.2 of the referred paper, concerning the Kerr-Vaidya space-time. The correct, new results, do support the claims in the paper.•1

•1: J: Cambiado

The calculations concerning the null normals and null expansions of the selected family of compact surfaces in the Kerr-Vaidya space-time were incorrect as presented in [1]. This affects several claims presented in section 4.2 of the paper. We correct the calculations and present the right results in this corrigendum.

The Kerr-Vaidya line-element can be written in advanced Eddington-Finkelstein coordinates[1] as:

$$\begin{aligned} ds^2 &= -\left(1-\frac{2M(u)r}{\rho^2}\right)du^2 + 2\ du\ dr + \rho^2 d\theta^2 - \frac{4aM(u)r\sin^2\theta}{\rho^2}d\phi\ du \\ &\quad -2a\sin^2\theta\ d\phi\ dr + \frac{(r^2+a^2)^2 - a^2\Delta\sin^2\theta}{\rho^2}\sin^2\theta d\phi^2, \end{aligned}$$

where $\rho^2 \equiv r^2 + a^2\cos^2\theta$ and $\Delta = \Delta(u,r) \equiv r^2 - 2rM(u) + a^2$. This is a non-vacuum solution of Einstein's field equations which contains matter violating the weak energy condition —unless $M(u) = M \neq 0$ is constant, in which case the metric reduces to that of the Kerr rotating black hole. For some discussion see [2, 3]. It is more difficult to discuss the black hole nature of the space-time in the generic case with non-constant $M(u)$, because it is not easy to find any dynamical horizon (or more generally, any marginally trapped tube) even though the space-time may contain closed (marginally) trapped surfaces.

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[1]The line-element in retarded Eddington-Finkelstein coordinates ($u \leftrightarrow -u$) was first published in [2].

We analyze the possibility of particle production associated with the surfaces $\mathcal{S}$ belonging to the family of compact surfaces (topologically S^2) given by constant values of the coordinates u and r. (For a different approach relating particle emission to the *event horizon*, see [4]). Two obvious normal one-forms are given by du and dr, and then the null normals, normalized with $l^+_\mu l^{-\mu} = -1$, can be given by

$$\underline{\mathbf{l}}^+ = \frac{\rho^2}{2\Theta^2}\left(-\Delta du + (r^2 + a^2 + \Theta)dr\right), \qquad \underline{\mathbf{l}}^- = -du + \frac{r^2 + a^2 - \Theta}{\Delta}dr$$

where Θ is a shorthand for

$$\Theta = \sqrt{(r^2 + a^2)\rho^2 + 2Mra^2\sin^2\theta}.$$

Observe that at the roots of $\Delta = 0$ one also has $r^2+a^2-\Theta = 0$ and thus the dr-component of $\underline{\mathbf{l}}^-$ can be well defined even if those roots exist (what happens whenever $M^2(u) \geq a^2$).

The mean curvature 1-form can be computed (e.g. using the formulas in [5]) to get

$$\underline{\mathbf{H}} = \frac{1}{\Theta^2}\left[[2r\rho^2 + a^2\sin^2\theta(M + r)]dr + a^2 r\dot{M}\sin^2\theta\, du\right]$$

where $\dot{M} = dM/du$ represents the derivative of $M(u)$. From here one can easily get the null expansions

$$\begin{aligned}
\theta^+ &= \frac{1}{2\Theta^3}\left[\Delta(2r\rho^2 + a^2\sin^2\theta(M + r)) + \dot{M}ra^2\sin^2\theta(r^2 + a^2 + \Theta)\right],\\
\theta^- &= -\frac{1}{\Theta\rho^2}\left[2r\rho^2 + a^2\sin^2\theta(M + r) + \frac{r^2 + a^2 - \Theta}{\Delta}\dot{M}ra^2\sin^2\theta\right]
\end{aligned}$$

and the dual expansion vector

$$\begin{aligned}
*\vec{H} &= \frac{1}{\rho^2\Theta}\left\{[2r\rho^2 + a^2\sin^2\theta(M + r)]\frac{\partial}{\partial u} - \dot{M}ra^2\sin^2\theta\frac{\partial}{\partial r}\right.\\
&\left.-\frac{a}{\Theta^2}\left[\rho^2\dot{M}ra^2\sin^2\theta - 2Mr[2r\rho^2 + a^2\sin^2\theta(M + r)]\right]\frac{\partial}{\partial\phi}\right\}. \qquad (1)
\end{aligned}$$

As we can see from the expression for $\theta^\pm$, the expansions cannot vanish on any entire surface in the family —they can vanish somewhere on the surface, that is, for some values of θ, but not elsewhere—, unless the surface lies on the intersection of $\Delta = 0$ with $\dot{M} = 0$. Thus, if there is a value $u = u_0$ such that

$$\dot{M}(u_0) = 0, \qquad M^2(u_0) \geq a^2$$

then the expansion θ^+ vanishes on the entire surfaces given by $u = u_0, r = r_\pm$, where $r_\pm$ are the roots of $\Delta(u_0, r) = 0$:

$$r_\pm = M(u_0) \pm \sqrt{M^2(u_0) - a^2}.$$

In general, these surfaces do not foliate any dynamical horizon (or marginally trapped tube), except for the Kerr case with $M =$ constant , in which case the hypersurface $\Delta = 0$ is the event horizon —and a Killing horizon.

For concreteness, we will focus on radiation from an outer MTS $(u_0,\ r_+)$ (although one can proceed in a similar manner for an inner MTS (u_0, r_-)). Since the surfaces defined by constant values of u and r are compact, we can write the effective surface gravity associated with these MTSs [1] as

$$\begin{aligned} \kappa &= -\sqrt{\frac{A(\mathcal{S}_+)}{16\pi}}\, l^{-\alpha}\nabla_\alpha \theta^+\rfloor_+ = \\ &= \frac{(r_+ - M)[4r_+^2\rho_+^2 + a^2(a^2+3r_+^2)\sin^2\theta] + a^4 r_+^2 \sin^4\theta\ \ddot{M}_+}{4r_+(a^2+r_+^2)^{3/2}\rho_+^2}, \end{aligned}$$

where the subscript '+' indicates that the quantity is evaluated on the MTS (u_0, r_+) and we have used that the area of the MTS is $A(\mathcal{S}_+) = 4\pi(r_+^2 + a^2)$.

The emission rate will then satisfy [1]

$$\Gamma \sim \exp\left(-8\pi \frac{r_+(a^2+r_+^2)^{3/2}\rho_+^2\omega_+}{(r_+ - M)[4r_+^2\rho_+^2 + a^2(a^2+3r_+^2)\sin^2\theta] + a^4 r_+^2\sin^4\theta\ \ddot{M}_+}\right). \tag{2}$$

Note that both the effective surface gravity and the emission rate in the general Kerr-Vaidya solution depend on the value of the coordinate θ.

Obtaining the *effective temperature* is straightforward by using the results in [1]. However, as argued in [1], to compute the *temperature* —associated with the radiation measured by one of the observers that detects it —requires identifying the specific observer and giving the function $M(u)$ explicitly.•2 To exemplify the procedure, we treat the simplest case with $M(u) = M$ =constant: the Kerr solution. Then $\vec{\xi}_u \equiv \partial/\partial u$ and $\vec{\xi}_\varphi \equiv \partial/\partial\varphi$ are Killing vectors providing two constants of motion along the null geodesics

•2: J:todo esto cambiado y acortado, y abajo, para no repetirnos

$$E = -S_{0,\alpha}\xi_u^\alpha = -S_{0,u} \quad \text{and} \quad J = S_{0,\alpha}\xi_\varphi^\alpha = S_{0,\varphi} \tag{3}$$

that notably simplifies dealing with them. We also choose, from our privileged observers (with unitary 4-velocity $\hat{\zeta}$ parallel to $*\vec{H}$), the particular ones at infinity. They measure an energy for a particle radiated from the MTS

$$\varepsilon = -S_{0,\alpha}\hat{\zeta}^\alpha\rfloor_\infty = E.$$

To identify the temperature T we take into account the statistical-mechanics fact that, according to these observers, the argument in the exponential of (2) should take the form $\alpha - \varepsilon/T$. By using the definition of ω in [1] together with (1) and (3) we get

$$\omega_+ = \frac{2r_+\rho_+^2 + a^2\sin^2\theta(M + r_+)}{2\rho_+^2\sqrt{a^2+r_+^2}}(E - \Omega_+ J), \tag{4}$$

where•3

•3: J: r_+ en vez de r_{H+}

$$\Omega_+ = \frac{a}{r_+^2 + a^2}$$

is the (constant) *angular velocity of the horizon* [6]. If we rewrite (2) using (4) we get

$$\Gamma \sim \exp\left[-\frac{\varepsilon - \Omega_+ J}{T}\right],$$

that allows us to get the temperature measured at infinity to be

$$T = \frac{\sqrt{M^2 - a^2}}{4\pi M(M + \sqrt{M^2 - a^2})}.$$

Note that this temperature does not depend of the angular variables. Both the emission rate and the temperature for this particular Kerr-metric case coincide with previous results in QFT [7].

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