Calculation of the ac to dc resistance ratio of conductive nonmagnetic straight conductors by applying FEM simulations

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Abstract
This paper analyzes the skin and proximity effects in different conductive nonmagnetic straight conductors' configurations subjected to applied alternating currents and voltages. These effects have important consequences, including a rise of the ac resistance, which in turn increases power loss, thus limiting the rating for the conductor. The alternating current (ac) resistance is important in power conductors and bus bars for line frequency applications as well as in smaller conductors for high frequency applications. Despite the importance of this topic, it is usually not analyzed in detail in undergraduate and even in graduate studies. For this purpose, this paper compares the results provided by available exact formulas for simple geometries with those obtained by means of two-dimensional finite element method (FEM) simulations and experimental results. The paper also shows that FEM results are very accurate and more general than those provided by the formulas, since FEM models can be applied in a wide range of electrical frequencies and configurations.

*Keywords***:** simulation, current density, ac resistance, finite element method, power loss.

1. INTRODUCTION

Copper and aluminum conductors are often used nowadays in many applications including in energy distribution systems and in electronics and communications devices such as telephones, televisions or computers among others [1]. It is well known that the current density distribution in any conductor carrying alternating current (ac) is often not uniform throughout the cross-section because of the skin and proximity effects [2]. At high frequencies the current tends to be concentrated towards the outer annulus of the conductor [3] and therefore the ac resistance can be much higher than the dc resistance, thus increasing power loss in the conductor [4]. The ac resistance greatly impacts the current rating of power cables [5] since it is limited by the maximum working temperature, which in turn affects the resistance. The ac resistance is an important performance parameter to optimally design any power cable [6].

Therefore, the calculation of the eddy currents effects and especially the ac resistance is of paramount importance [7]. However, analytical methods to deal with this problem are quite limited since they are only available for some simple geometries [8] mainly due to the complexity of this problem since it requires solutions to solve Maxwell's equations in inhomogeneous unbounded regions [9]. Consequently, numerical methods are appealing to analyze in detail such types of problems, since they are more flexible [8]. The finite element method (FEM) has been applied to accurately calculate power losses in power cables, inductors and transformer windings with different geometries [10-12]. The purpose of this paper is to take advantage of the simplicity and flexibility of FEM simulations to analyze skin and proximity effects in different configurations for the conductors, which include systems with one or more than one conductor with different cross sections. In addition, FEM results are compared with those provided by exact analytical solutions found in the technical literature (when available) and with published experimental results. The numerical approach presented in this paper can be useful for both undergraduate and graduate physics and

engineering courses. It is worth noting that the conductors considered in this paper are assumed to have uniform cross section and uniform nonmagnetic material properties, and to be straight and infinitely long.

2. SKIN AND PROXIMITY EFFECT IN STRAIGHT CONDUCTORS

Due to the skin effect, the current density within a conductor carrying an alternating current (ac) is largest near the conductor surface and decreases towards the center [3]. Assume a semi-infinite good conductor with uniform electric conductivity σ and magnetic permeability $\mu = \mu_r \cdot \mu_0$ occupying the space $z > 0$ as shown in Fig. 1 [13].
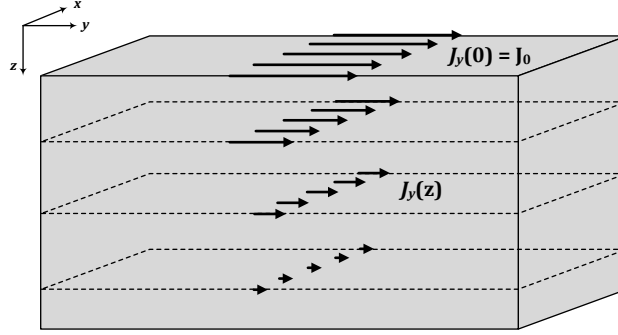
Fig. 1. Semi-infinite homogeneous good conductor in which a sinusoidal current density J_y is directed towards the y direction.

Consider a sinusoidal current density $J_y(z)$ with angular frequency $\omega = 2\pi f$ directed towards the y direction. It will be shown that the intensity of the current density vector is attenuated towards the z direction.

Under quasi-static conditions, that is, when the system dimensions are small compared with the associated electromagnetic wavelength [14], the displacement current in a good conductor can be neglected. Therefore, (1) and (2) are derived from the Maxwell equations,

$$\vec{\nabla} \wedge \vec{E} = -\partial \vec{B} / \partial t = -j\omega \vec{B} \tag{1}$$

$$\vec{\nabla} \wedge \vec{H} = \vec{J} \tag{2}$$

$\vec{E}, \vec{B}$ and $\vec{H}$ being, respectively, the electric field vector, the magnetic induction field and the magnetic field vectors.

Since $\vec{E} = \vec{J}/\sigma$ and $\vec{H} = \vec{B}/\mu$, (1) and (2) can be rewritten as,

$$\vec{\nabla} \wedge \vec{J} = -j\omega\sigma \vec{B} \tag{3}$$

$$\vec{\nabla} \wedge \vec{B} = \mu \vec{J} \tag{4}$$

Due to the symmetry conditions of this problem, the magnetic induction field is directed towards the x direction, that is, $\vec{B}(z) = (B_x, 0, 0)$ where $B_x = B_x(z)$. Since the current density decreases towards the z axis, the curl operators in (3) and (4) lead to,

$$dJ_y / dz = j\omega\sigma B_x \tag{5}$$

$$dB_x / dz = \mu J_y \tag{6}$$

Finally, by combining (5) and (6), the current density J_y satisfies,

$$d^2 J_y / dz^2 = j\omega\mu\sigma J_y \tag{7}$$

whose solution is,

$$J_y(z) = A e^{z\sqrt{j\omega\mu\sigma}} + B e^{-z\sqrt{j\omega\mu\sigma}} \tag{8}$$

Where $\sqrt{j\omega\mu\sigma} = (1+j)\sqrt{\omega\mu\sigma/2} = (1+j)\sqrt{\pi f \mu\sigma} = (1+j)k$ and

$$\delta = 1/k = 1/\sqrt{\pi f \mu\sigma} \tag{9}$$

is the skin depth, that is the distance z at which the intensity of the current density falls to 36.8% (1/e) of

its value J_0 at the interface $z = 0$. From (9) it is deduced that the skin depth decreases with increasing values of the frequency, permeability and material conductivity.

By applying the boundary conditions $J_y(z=0) = J_0$ and $J_y(z\rightarrow \infty) = 0$, (8) results in,

$$J_y(z) = J_0 e^{-kz} e^{-jkz} \tag{10}$$

which shows that the intensity of the current density is attenuated towards the z direction. Although the analysis to obtain (10) has been carried out for a specific geometry, this result can be generalized for conductors in which the external diameter is much larger than the skin depth [13].

The effect described by (10) is known as skin effect, being especially important at higher frequencies where the skin depth is smaller. The skin effect reduces the effective cross-section of the conductor, thus increasing the effective or ac resistance of the conductor and has important practical consequences [1]. The skin effect is produced by the eddy currents induced by the magnetic field generated by the alternating current. The ac magnetic field associated with the electric current flowing through the conductor induces an electromotive force (EMF) in the interior of the conductor which induces eddy currents inside the conductor. These currents partially cancel out the conduction current in the center of the conductor, thus reinforcing the conduction current in the skin of the conductor, as shown in Fig. 2. Therefore the conducting electrons tend to flow in the outer layers of the conductor. The irregular current density in the interior of the conductor tends to increase its ac resistance [5] since the effective cross-sectional area is reduced.
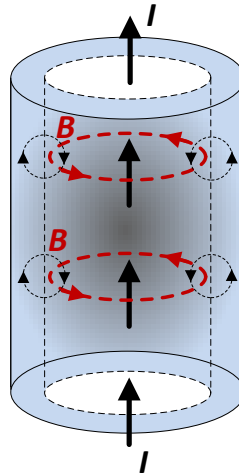
Fig. 2. Skin effect as a consequence of the induced eddy currents. It is due to the circulating eddy currents (arising from a changing B field) cancelling out the current flow in the center of the conductor and reinforcing it in the skin. The currents are in black color whereas the magnetic field density is in red color.

Fig. 3 shows the current density decay due to the skin effect resulting from FEM simulations.
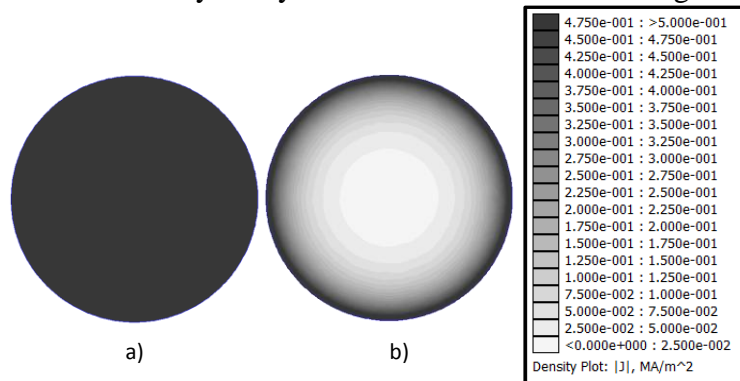
Fig. 3. Skin effect in an AWG-0000 solid round conductor (11.68 mm diameter) of copper carrying 10 A. Current density plot obtained by means of FEM simulations. a) 50 Hz b) 5 kHz.

To evaluate the R_{ac}/R_{dc} resistance ratio in a conductor, it is first necessary to determine its dc resistance, which is calculated as [15],

$$R_{dc} = \frac{\rho_{20}}{S}[1+\alpha_{20}(T-T_0)] \ [\Omega/m] \tag{11}$$

ρ_{20} being the resistivity at $T_0 = 20$ °C, α_{20} is the temperature coefficient at 20 °C and S the cross-sectional area of the conductor.

The skin effect has important practical consequences when dealing with large cross-sectional conductors at power frequency, since most of internal part of the conductor carries very little current. In this case tubular conductors can be used, thus reducing the amount of material, weight and cost. Round tubular conductors are used in different type of applications, including substations bus bars [16], particle accelerators, magnetic resonance image devices or induction furnaces among others. In these applications a wide range of supply frequency is applied.

When dealing with two or more nearby conductors, the ac magnetic field generated by each conductor induces eddy currents in the others, thus influencing the current density distribution in the nearby conductors. It can be understood as an inductive coupling among the nearby conductors. This is the proximity effect, which is more pronounced at high frequencies and small axial spacing between conductors, thus becoming prominent when dealing with very close conductors. Therefore, in each conductor there are self-induced eddy currents (skin effect) and eddy currents induced by the nearby conductors (proximity effect) [13]. The proximity effect alters the current density distribution within the conductor, thus increasing in some regions of the conductors while diminishing in other regions. It increases the effective ac resistance since the current is confined in a reduced area of the conductor, thus raising power losses when compared to those in isolated conductors. The proximity effect has great impact on all kind of windings in electrical machines such as transformers, electrical motors and generators among others and it is of especial importance in switch-mode power supplies [17].

The mathematical description of the proximity effect is more complex than that of the skin effect and an exact solution is only attainable in a very few geometries. However, when considering two parallel nearby conductors a qualitative description is possible. According to Lenz's law, the magnetic field generated by the alternating current in each conductor induces eddy currents in the other, as shown in Fig. 4. According to Ampere's law, the magnitude of the induced eddy currents decreases when increasing the spacing between the conductors. As shown in Fig. 4, when two parallel solid conductors carry currents of the same polarity, the current density decreases in the conductors' sides facing each other while increasing in the sides farthest away from both conductors. Contrarily, when the currents have opposite polarity, the current density increases in the adjacent conductors' sides, but it decreases in the remote conductors' sides. This effect is also important in insulated cables operated at power frequency since they can be placed very close together [18].
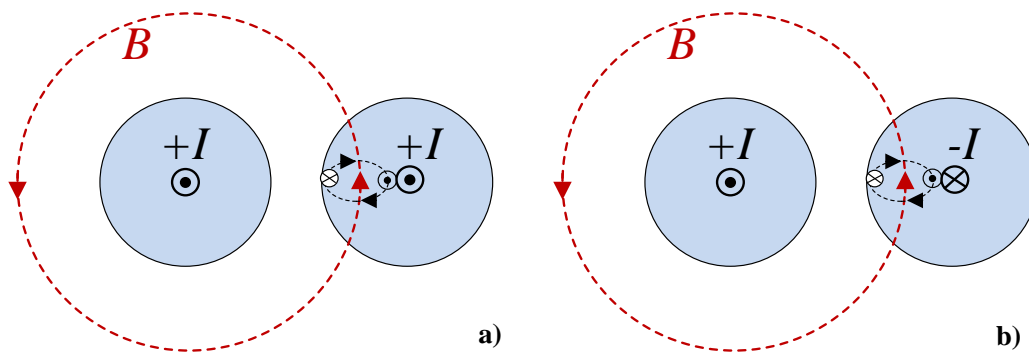
Fig.4. Proximity effect between two parallel solid round conductors. For simplicity only the effect of the left conductor on the right one is shown. a) Conductors with same polarity. b) Conductors with opposite polarity. The currents are in black color whereas the magnetic field density is in red color.

Fig. 5 shows the combined effects of the skin and proximity effects in two parallel AWG-0000 (11.68 mm diameter) solid round conductors of copper, spaced 0.3 mm and carrying 10 A each, when carrying currents with the same and opposite polarities.
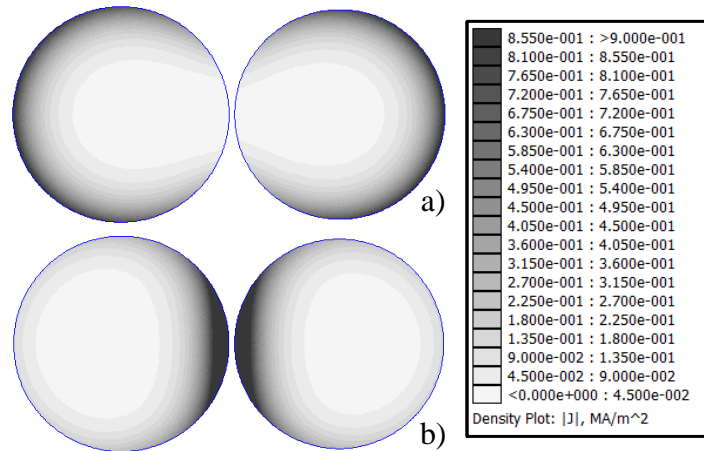

Fig. 5. Skin and proximity effects at 5 kHz in two AWG-0000 (11.68 mm diameter) solid round conductor of copper spaced 0.3 mm carrying 10 A each. Current density plot obtained by means of FEM simulations. a) Same polarity, +10A and +10A. b) Opposite polarity, +10A and -10A.

The mathematical theory to solve this type of problems has been mainly based on the integral equation approach [19-24]. However, the solution of these equations usually offers challenging mathematical difficulties, thus the solution of a particular skin or proximity effect problem often involves applying an approximate numerical procedure based on an iterative solution of a set of algebraic equations. Exact solutions only exist for a few geometries including isolated straight round [3,4,25] and tubular conductors [26].

3. RESULTS

Finite element modelling (FEM) is recognized as a useful tool to perform realistic simulations of complex problems, since it can provide results accurate enough to compute the unequal current distribution in conductors due to the skin and proximity effects [27,28]. In this paper the free license two-dimensional FEMM package [29] has been used since it is well suited for this application.

Fig. 6 compares the R_{ac}/R_{dc} resistance ratio of a single solid copper conductor by means of the results provided by FEM simulations and the exact formula found in [3,4,25] with those from experimental data found in [30]. This comparison is done for an AWG-12 copper round conductor (2.052 mm diameter) in the range 0-100 kHz.
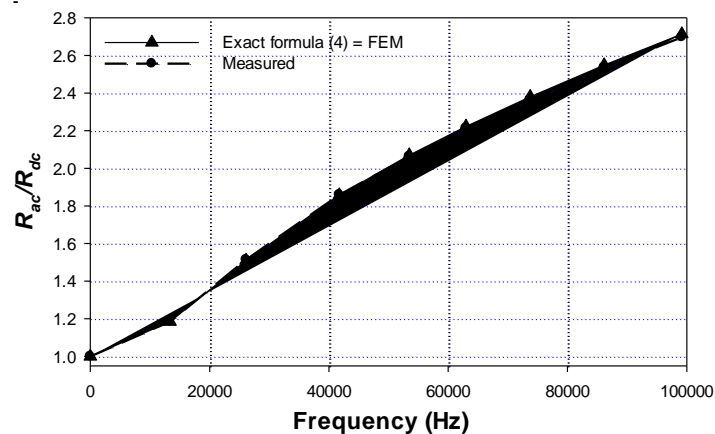
Fig. 6. Skin effect in an isolated round solid annealed AWG-12 copper conductor with diameter 2.052 mm between 0 and 100 kHz. Comparison between results provided by the exact formula, FEM simulations and experimental data.

Results presented in Fig. 6 clearly show the accuracy of the FEM method when compared to the results provided by the exact formula and with experimental results. Therefore it is concluded that the FEM model can accurately predict the R_{ac}/R_{dc} ratio for this geometry.

Fig. 7 compares the R_{ac}/R_{dc} resistance ratio for two AWG-0000 (11.68 mm diameter) parallel round solid conductors spaced 0.3 mm and 200 mm when the copper conductors carry ac currents with the same and opposite polarities. Results compared in Fig. 8 are provided by the FEM method and by experimental results from [31].
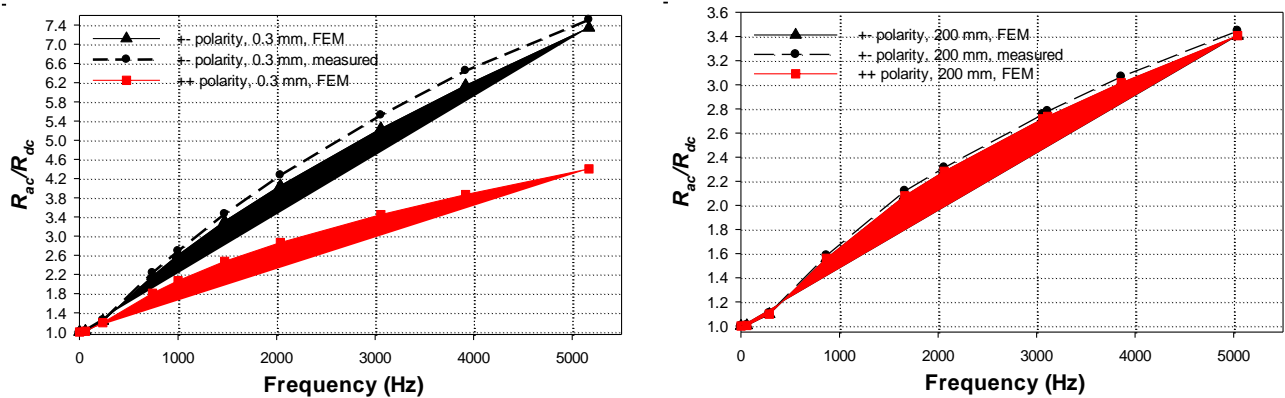
Fig. 7. Skin and proximity effects in two parallel round solid conductors. Effect of current polarity analyzed from FEM simulations. a) Two AWG-0000 (11.68 mm diameter) parallel solid conductors spaced 0.3 mm. b) Conductors spaced 200 mm.

FEM results shown in Figs. 7 prove that when dealing with two parallel conductors, the polarity of the currents can significantly influence the R_{ac}/R_{dc} ratio, especially when both conductors are very close each other. It is also proven that the combination of both skin and proximity effects can greatly increase power losses in conductors, since the R_{ac}/R_{dc} ratio increases when the spacing between conductors decreases. This asymmetric behavior is produced because in the case of two conductors carrying currents of opposite polarity the current density is concentrated in the conductors' sides facing the other conductor, thus being increased when the conductors are very close each other. Contrarily, when the conductors carry currents of the same polarity, the current density is concentrated in the regions farthest away from both conductors, so they are less affected by the distance between conductors.

Fig. 8 shows the effect of the axial spacing between parallel conductors. It shows a reduction of the resistance ratio when increasing the axial spacing between conductors. These results clearly indicate that when the spacing is large enough, the resistance ratio tends to the value of an isolated conductor since the proximity effect becomes negligible.
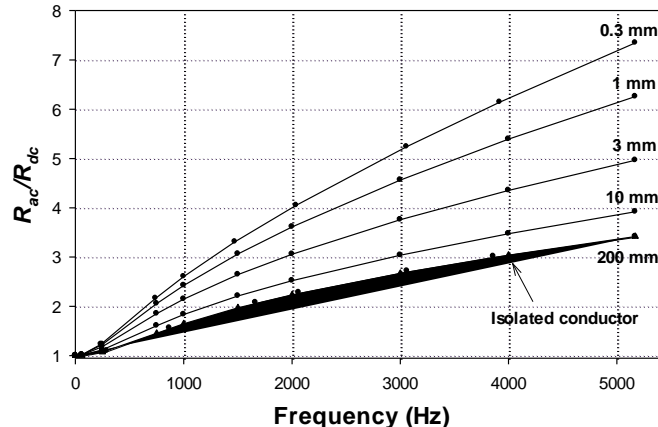
Fig. 8. Skin and proximity effects in two AWG-0000 (11.68 mm diameter) parallel round solid conductors carrying currents of opposite polarity. Effect of the axial spacing between conductors (0.3 mm to 200 mm).

Fig. 9 shows the ac resistance of a copper round tubular conductor of fixed outer radius $r_2 = 10$ mm and variable inner radius r_1 calculated at 5000 Hz. At this frequency the skin depth is about 1 mm, so when the wall thickness is larger than 4 mm, that is, about four times the skin depth, the central part of the conductor almost does not carry any current. This effect is deduced from Fig. 9, since for wall thicknesses above 4 mm, the ac resistance remains almost constant, so the central part of the conductor could be removed without affecting the current carrying capacity and power losses.
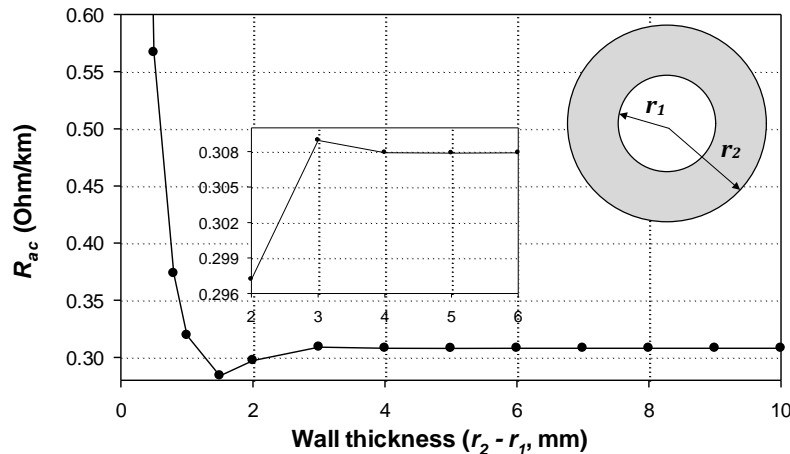
Fig. 9. FEM simulations. Skin effect in a round tubular conductor with fixed outer radius $r_2 = 10$ mm. Effect of the wall thickness when varying the inner radius r_1 from 0 mm to 9.5 mm.

4. CONCLUSIONS

This paper has analyzed the R_{ac}/R_{dc} ratio in different configurations of conductors by applying FEM simulations. These results have been compared with those obtained from published exact formulas, when available, and with those from recognized experimental works. Such types of experimental measurements are time-consuming and require expensive equipment, whereas the validity of the analyzed approximated formulas is restricted to some simple geometries. However, FEM simulations have shown excellent accuracy and flexibility since they allow analyzing a wide range of electrical frequencies and geometries. This simulation method can be used to develop practical sessions in lecture demonstrations or computer assisted laboratory experiments in different physics, or engineering undergraduate or graduate courses.

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