

# Footstep Parameterized Motion Blending using Barycentric Coordinates

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## Abstract

This paper presents a real-time animation system for fully-embodied virtual humans that satisfies accurate foot placement constraints for different human walking and running styles. Our method offers a fine balance between motion fidelity and character control, and can efficiently animate over sixty agents in real time (25 FPS) and over a hundred characters at 13 FPS. Given a point cloud of reachable support foot configurations extracted from the set of available animation clips, we compute the Delaunay triangulation. At runtime, the triangulation is queried to obtain the simplex containing the next footstep, which is used to compute the barycentric blending weights of the animation clips. Our method synthesizes animations to accurately follow footsteps, and a simple IK solver adjusts small offsets, foot orientation, and handles uneven terrain. To incorporate root velocity fidelity, the method is further extended to include the parametric space of root movement and combine it with footstep based interpolation. The presented method is evaluated on a variety of test cases and error measurements are calculated to offer a quantitative analysis of the results achieved.

*Keywords:*

Character animation, Crowd simulation, Footsteps controller

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## 1. Introduction

Crowd simulation research has matured in recent years with important applications in training, building design, psychological studies, and video-games. All these applications benefit from having fully-embodied virtual human characters animated in real-time while accurately satisfying control objectives without any noticeable artifacts.

Algorithms that generate center of mass (COM) trajectories [1, 2, 3, 4] lead to ambiguities when trying to superimpose a fully articulated virtual human to follow them, thus producing foot-sliding artifacts when no suitable animation is found, or when the root orientation and the displacement vector of the animation do not match. Different animations can be blended by tweaking some of the upper body joints [5] to minimize artifacts, at the expense of constant updates to account for the decoupling between the crowd simulation and the animation system.

Footstep-based control systems [6, 7] output a list of space-time foot-plants to define a fine-grained trajectory with fewer ambiguities that can solve more complex scenarios (e.g., complex manipulation tasks requiring careful control of the lower body, or collaborative tasks, such as careful sidestepping to make way for another agent in a narrow corridor). To realistically represent such simulations, we need a method to synthesize animations that accurately follow the output trajectory, i.e., accurate placement of feet with space-time constraints. This problem is traditionally known as the *stepping stone* problem.

Moreover, the output trajectory can be modified by external perturbations such as uneven terrain.

We present an online animation synthesis technique for fully embodied virtual humans that satisfies foot placement constraints for a large variety of locomotion speeds and styles (see Fig. 1). Given a database of motion clips, we precompute multiple parametric spaces based on the motion of the root and the feet. A root parametric space is used to compute a weight for each available animation based on root velocity. Two foot parametric spaces are based on a Delaunay triangulation of the graph of possible foot landing positions. For each foot parametric space, blending weights are calculated as the barycentric coordinates of the next footstep position for the triangle in the graph that contains it. These weights are used for synthesizing animations that accurately follow the footstep trajectory while respecting the singularities of the different walking styles captured.

Blending weights calculated as barycentric coordinates are used to reach the desired foot landing by interpolating between several proximal animations, and IK is used to adjust the final position of the support foot to correct for minor offsets, foot step orientation and the angle of the underlying floor.

Since foot parametric space only considers final landing positions of the feet without taking into account root velocity, this may lead to the selection of animations that satisfy position constraints but introduce discontinuities in root velocity. To incorporate root velocity fidelity we present a method that can integrate both foot positioning and root velocity fidelity. Our method also allows the system to recover nicely when the input

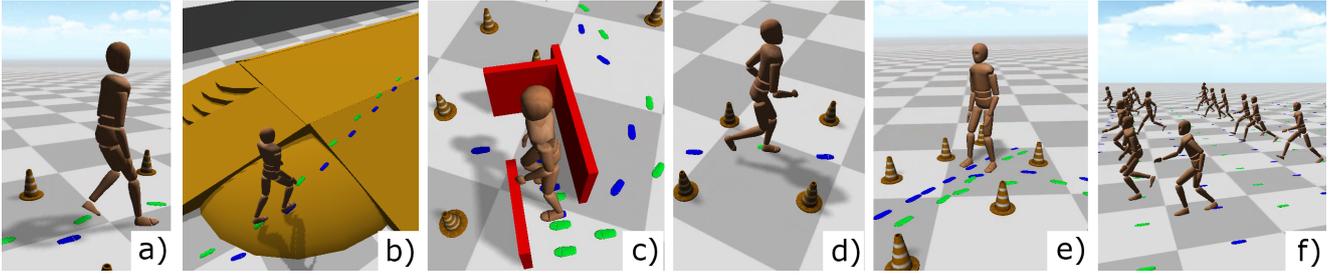


Figure 1: An autonomous virtual human navigating a challenging obstacle course (a), walking over a slope (b), exercising careful foot placement constraints including side-stepping (c), speed variations (d), and stepping back (e). The system can handle multiple agents in real time (f).

56 foot trajectory contains steps that are not possible to perform  
 57 with the given set on animations (for example, due to extreme  
 58 distance between steps).

59 The presented method is evaluated on a variety of test cases  
 60 and error measurements are calculated to offer a quantitative  
 61 analysis of the results achieved. Our framework can efficiently  
 62 animate over sixty agents in real time (25 FPS) and over a hun-  
 63 dred characters at 13 FPS, without compromising motion fi-  
 64 delity or character control, and can be easily integrated into ex-  
 65 isting crowd simulation packages. We also provide the user  
 66 with control over the trade-off between footstep accuracy and  
 67 root velocity.

## 68 2. Related Work

69 Locomotion synthesis can be tackled from different points  
 70 of view depending on how the character is being controlled.  
 71 If a user controls the character with a 3rd person controller, it  
 72 is common to work on a root velocity basis, because the user  
 73 wants to move the character around in an agile way. In such  
 74 cases, like video-games, real-time response is critical and arti-  
 75 facts such as foot skating can be ignored. Optimization based  
 76 approaches [8] are able to synthesize animations that conform  
 77 to velocity and orientation constraints. However, they need a  
 78 very large database and their computational time does not al-  
 79 low many characters in real-time. Semi-procedural animation  
 80 systems [9] work with a small set of animations and use inverse  
 81 kinematics only over the legs to ensure ground contact and to  
 82 adapt the feet to possible slopes of the terrain, but they are un-  
 83 able to follow footstep trajectories.

84 Animation systems for autonomous agents must be com-  
 85 putationally efficient to animate a multitude of characters in  
 86 real-time, and need to follow different control trajectories, de-  
 87 pending on the controller used. Controllers that account for an-  
 88 imation constraints while computing control decisions such as  
 89 motion graphs [10, 11, 12, 13] or precomputed search trees [14]  
 90 can simply playback the animation sequence. These approaches  
 91 try to reach the goal by connecting series of motion[15], which  
 92 sometimes limits the movements of the agents. The main issues  
 93 with motion graphs are that they require a very large amount  
 94 of animation clips (over 400) and have a high computational  
 95 cost which makes them not suitable for large groups of agents  
 96 in real-time. Precomputed search trees can handle groups, but

97 work with a few animation clips and are unable to synthesize  
 98 new animations.

99 Approaches that ignore animation constraints produce center  
 100 of mass trajectories for the animation system to follow. Dif-  
 101 ferent models include social forces [2], rule-based approaches [1],  
 102 flow tiles [16], roadmaps [17], continuum dynamics [3], and  
 103 force models parametrized by psychological and geometrical  
 104 rules [4]. These techniques can easily simulate hundreds and  
 105 thousands of characters in real-time, but do not account for  
 106 locomotion constraints, thus producing artifacts such as foot-  
 107 sliding which require correction and simulation updates [5].

108 Considering the root velocity as the input parameter for  
 109 character control, numerous approaches can synthesize smooth,  
 110 versatile and more plausible locomotion animations [18, 9]. Some  
 111 approaches have also used the idea of selecting animations from  
 112 a Delaunay triangulation of all the available animation clips  
 113 [19, 20]. But all of these approaches are restricted to the root  
 114 for performing character control.

115 There has been a recent surge in approaches that produce  
 116 footstep trajectories for character control. They can be phys-  
 117 ically based but generated off-line [21], be generated online  
 118 from an input path computed by a path planner [6], or use sim-  
 119 plified control dynamics to produce bio-mechanically plausi-  
 120 ble footstep trajectories for crowds [7]. These approaches of-  
 121 ten show their animation results off-line using tools such as 3D  
 122 Max [22].

123 Footstep-driven animation systems [23] produce unnatural  
 124 results using procedural methods. The work in [24] uses a sta-  
 125 tistical dynamic model learned from motion capture data in ad-  
 126 dition to user-defined space-time constraints (such as footsteps)  
 127 to solve a trajectory optimization problem. In [25] random  
 128 samples of footsteps make a roadmap going from one point  
 129 to another which is used to find a minimum-cost sequence of  
 130 motions matching it and then retarget to the exact foot place-  
 131 ments. The work in [26, 27] performs a global optimization  
 132 over an extracted center of mass trajectory to maximize the  
 133 physical plausibility and perceived comfort of the motion, in  
 134 order to satisfy the footprint constraints. Recent solutions [6,  
 135 28, 29] adopt a greedy nearest-neighbor approach over larger  
 136 motion databases. To ensure spatial constraints, the character  
 137 is properly aligned with the footsteps and reinforced with in-  
 138 verse kinematics, while temporal constraints are satisfied us-  
 139 ing time warping. These techniques achieve highly accurate re-  
 140 sults in terms of foot positioning, but their computational cost

141 makes them unsuitable for real-time animation of large groups  
142 of agents.

143 **Comparison to Prior Work.** Our method produces visually  
144 appealing results with foot placement constraints, using only a  
145 handful of motion clips, and can seamlessly follow footstep-  
146 based control trajectories while preserving the global appear-  
147 ance of the motion. Compared to [9], we exploit the combi-  
148 nation of multiple parameter spaces for footstep-precision con-  
149 trol. This reduces the dimensionality of the problem, compared  
150 to [29]. Unlike previous work in the literature, our method can  
151 synthesize animations for a large number of characters in real  
152 time, following footstep trajectories for different walking styles  
153 and even running motions with a small flying phase.

### 154 3. Framework Overview

155 Animating characters in real time animations has different  
156 requirements depending on the application. In many applica-  
157 tions, the user only wants to control the direction of movement  
158 and speed of the root, but there are other situations where a  
159 finer control of the foot positioning is required. For example,  
160 the user may want to respect different walking gaits depending  
161 on the terrain, to make the character step over stones to cross  
162 a river, or walk through some space full of holes whilst avoid-  
163 ing falling. For this purpose we have developed a framework to  
164 animate virtual characters following footstep trajectories, while  
165 still being able to follow trajectories based on the movement of  
166 the COM when necessary.

167 Online locomotion systems [9] traditionally produce syn-  
168 thesized motions that follow a COM trajectory, with procedural  
169 corrections for uneven terrain. These methods can nicely fol-  
170 low COM trajectories, but they lack control over the style of  
171 walking and the kind of steps. For instance, we cannot control  
172 whether in order to walk fast, the character will move with large  
173 distances between steps or with a fast sequence of short steps.  
174 This is the main issue we address in our work: to provide an  
175 animation system that is able to accurately follow footstep tra-  
176 jectories while meeting real-time constraints, and that can scale  
177 to handle large groups of animated characters.

178 For this purpose, we introduce two parametric spaces based  
179 on the position of each foot:  $\Omega_{f_L}$  and  $\Omega_{f_R}$ , and switch between  
180 the two depending on the swing foot, as well as a parametric  
181 space based on the root movement  $\Omega_{f_R}$ . Our technique takes  
182 into account both displacement (from  $\Omega_{f_L}$  and  $\Omega_{f_R}$ ) and speed  
183 (from  $\Omega_r$ ) to ensure the satisfaction of both spatial and temporal  
184 constraints. Our system provides the user with the flexibility  
185 to choose between different control granularities ranging from  
186 exact foot positioning to exact root velocity trajectories. Fig. 2  
187 shows our framework.

### 188 4. Footstep-based Locomotion

189 The main goal of the Footstep-based Locomotion Controller  
190 is to accurately follow a footstep trajectory, i.e., to animate a  
191 fully articulated virtual human to step over a series of foot-  
192 plants with space and velocity constraints. The system must

193 meet real-time constraints for a group of characters, should be  
194 robust enough to handle sparse motion clips, and needs to pro-  
195 duce synthesized results that are void of artifacts such as foot  
196 sliding and collisions.

#### 197 4.1. Motion Clip Analysis

198 From a collection of cyclic motion clips<sup>1</sup>, we need to extract  
199 individual footsteps. Each motion clip contains two steps, one  
200 starting with the left foot on the floor, and one starting with  
201 the right foot. A step is defined as the action where one foot  
202 of the character starts to lift-off the ground, moves in the air  
203 and finishes when it is again planted on the floor. We say that  
204 a footstep corresponds to one foot when that foot is the one  
205 performing the action previously described. The foot that stays  
206 in contact with the floor for most of the duration of the footstep  
207 is called the supporting foot, since it supports the weight of the  
208 body. This applies even for running motions, where the support  
209 foot goes into fly mode for a short phase of the footstep, but it is  
210 still the one supporting the weight during most of the footstep.

211 During an offline analysis, each motion clip  $m_i$  is annotated  
212 with the following information: (1)  $\mathbf{v}_i^r$ : Root velocity vector. (2)  
213  $\mathbf{d}_i^L$ : Displacement vector of the left foot. (3)  $\mathbf{d}_i^R$ : Displacement  
214 vector of the right foot.

215 Similar to [9], animations are analyzed in place, that is, we  
216 ignore the original root forward displacement, but keep the ver-  
217 tical and lateral deviations of the motion. This allows an auto-  
218 matic detection of foot events, such as lifting, landing or plant-  
219 ing, from which we can deduce the displacement vector of each  
220 foot. For example, the displacement vector of the left foot  $\mathbf{d}_i^L$   
221 is obtained by subtracting the right foot position at the instant  
222 of time when the left foot lands, from the right foot position  
223 at the instant of time when the left foot is lifting off. These  
224 displacements will be later used to move the whole character,  
225 eliminating any foot sliding. By adding  $\mathbf{d}_i^L$  to  $\mathbf{d}_i^R$  and knowing  
226 the time duration of the clip, we can calculate the average root  
227 velocity vector  $\mathbf{v}_i^r$  of the clip  $m_i$ .

228 This average velocity is used to classify and identify ani-  
229 mations, by providing an example point which is the input  
230 for the polar gradient band interpolator (where each example  
231 point represents a velocity in a 2D parametric space). Gradi-  
232 ent band interpolation specifies an influence function associ-  
233 ated with each example, which creates gradient bands between  
234 the example point and each of the other example points. These  
235 influence functions are normalized to get the weight functions  
236 associated with each example. However the standard gradient  
237 band interpolation is not well suited for interpolation of exam-  
238 ples based on velocities. The polar gradient band interpolation  
239 method is based on reasoning that in order to get more desir-  
240 able behavior for the weight functions of example points that  
241 represent velocities, the space in which the interpolation takes  
242 place should take on some of the properties of a polar coordi-  
243 nate system. It allows for dealing with differences in direction

<sup>1</sup>Although cyclic animations are not strictly required by our method, they help find smoother transitions between consecutive footsteps and are preferred by most standard animation systems [9].

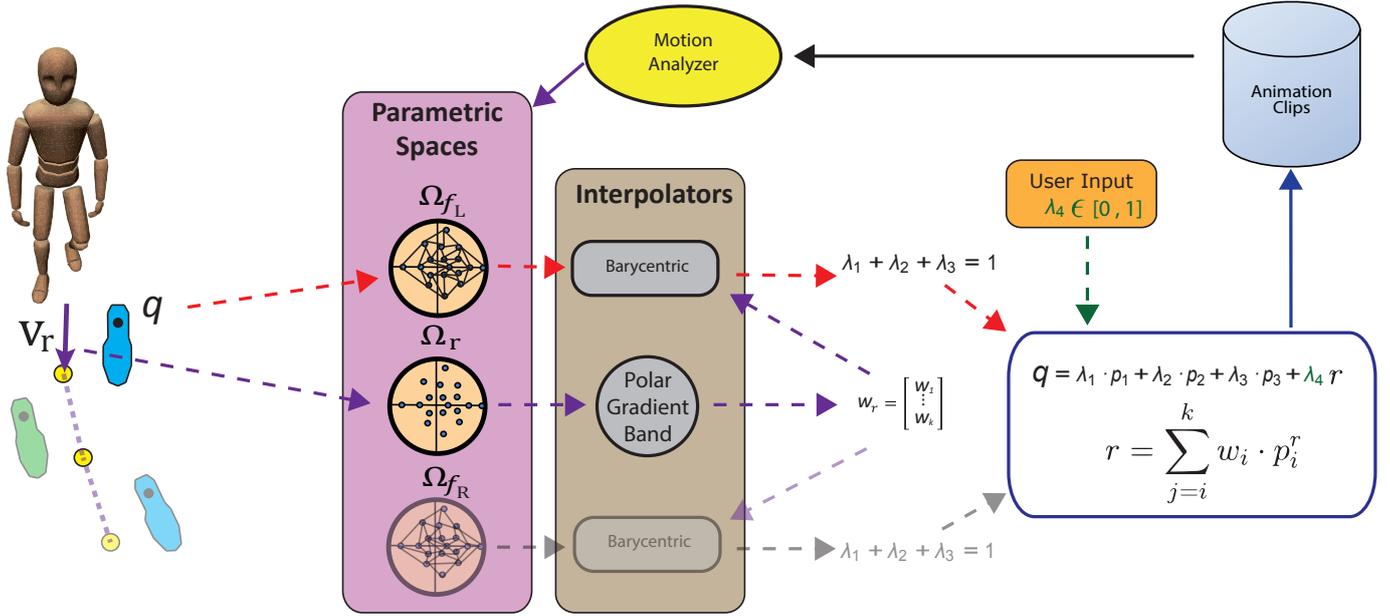


Figure 2: Online selection of the blend weights to accurately follow a footstep trajectory.  $\Omega_r$  uses a gradient band polar based interpolator [9] to give a set of weights  $w_j$ , which are then used by the barycentric coordinates interpolator to tradeoff between footstep and COM accuracy.

244 and magnitude rather than differences in the Cartesian vector  
 245 coordinate components. For more details we refer the reader to  
 246 [9].

247 Each motion clip is then split into two animation steps  $A_i^L$   
 248 for the left foot and  $A_i^R$  for the right foot. For each foot, we need  
 249 to calculate all the possible positions that can be reached based  
 250 on the set of animation steps available. Since the same analysis  
 251 is performed for both feet separately, from now on we will not  
 252 differentiate between left and right for the ease of exposition.  
 253 For each individual step animation  $A_i$  and given an initial root  
 254 position, we want to extract the foot landing position  $p_i$ , if the  
 255 corresponding section of its original clip was played. This is  
 256 calculated by summing the root displacement during the section  
 257 of the animation with the distance vector between the root  
 258 projection over the floor and the foot position in the last frame.

259 The set  $\{p_i | \forall i \in [1, n]\}$  where  $n$  is the number of step ani-  
 260 mations, provides a point cloud. Fig. 3 shows the Delaunay  
 261 triangulation that is calculated for the point cloud of landing  
 262 positions. This triangulation is queried in real time to deter-  
 263 mine the simplex that contains the next footstep in the input  
 264 trajectory. Once the triangle is selected, we will use its three  
 265 vertices  $p_1$ ,  $p_2$  and  $p_3$  to compute the blending weights for each  
 266 of the corresponding animations  $A_1$ ,  $A_2$  and  $A_3$ .

#### 267 4.2. Footstep and Root Trajectories

268 Our system can work with both footstep trajectories and  
 269 COM trajectories. A footstep trajectory will be given as an ordered  
 270 list of space-time positions with orientations, whether it  
 271 is precomputed or generated on-the-fly.

272 The input footstep trajectory may be accompanied by its as-  
 273 sociated root trajectory (a space-time curve, rather than a list of  
 274 points, and an orientation curve), or else we can automatically  
 275 compute it from the input footsteps by interpolation. This is

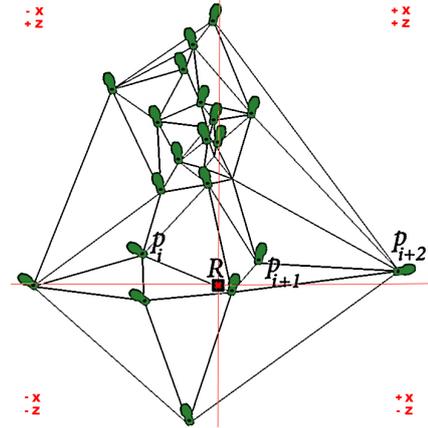


Figure 3: Delaunay triangulation for the vertices representing the landing positions ( $p_i, p_{i+1}, p_{i+2}, \dots$ ) of the left foot when the root,  $R$ , is kept in place.

276 achieved by computing the projection of the root on the ground  
 277 plane, as the midpoint of the line segment joining two consecu-  
 278 tive footsteps. The root orientation is then computed as the  
 279 average between the orientation vectors of each set of consecu-  
 280 tive steps. This provides us with a sequence of root positions  
 281 and orientations which can be interpolated to approximate the  
 282 motion of the root over the course of the footstep trajectory.

#### 283 4.3. Online Selection

284 During run time, the system animates the character towards  
 285 the current target footstep. If the target is reached, the next foot-  
 286 step along the trajectory is chosen as the next target. For each  
 287 footstep  $q_j$  in the input trajectory  $\{q_1, q_2, q_3, \dots, q_m\}$  we need to  
 288 align the Delaunay triangulation graph with the current root posi-  
 289 tion and orientation. Then the triangle containing the next foot

290 position is selected as the best match to calculate the weights re-  
 291 quired to nicely blend between the three animations in order to  
 292 achieve a footstep that will land as close as possible to the de-  
 293 sired destination position  $q_j$  (Fig. 4). Notice that these weights  
 294 are applied equally to all the joints in the skeleton, which means  
 295 that at this stage we cannot accurately adjust the specific foot  
 296 orientation required by each footstep in the input trajectory.

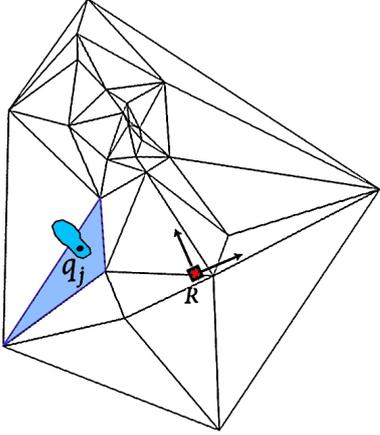


Figure 4: By matching root position and orientation, we can determine the triangle containing the destination position for the landing position  $q_j$ .

#### 297 4.4. Interpolation

298 Footstep parameters change between successive footplants,  
 299 remaining constant during the course of a single footstep (sev-  
 300 eral frames of motion). Therefore we need to compute the best  
 301 interpolation for each footstep, blend smoothly between con-  
 302 secutive steps, and apply the right transformation to the root in  
 303 order to avoid foot-sliding or intersections with the ground.

304 To meet these requirements, we use a barycentric coordi-  
 305 nates based interpolator in  $\Omega_{f_L}$  and  $\Omega_{f_R}$ , and constrain the so-  
 306 lution based on the weights computed in  $\Omega_r$ . This allows us to  
 307 animate a character at the granularity of footsteps, while simul-  
 308 taneously accounting for the global motion of the full body.

309 If we only consider the footstep parametric space, then the  
 310 vertices of the selected triangle are those that can provide the  
 311 best match for the desired foot position. The barycentric co-  
 312 ordinates of the desired footstep are calculated for the selected  
 313 triangle as the coordinates that satisfy:

$$q_j = \lambda_1 \cdot p_1 + \lambda_2 \cdot p_2 + \lambda_3 \cdot p_3, \quad (1)$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 1$$

314 where  $p_1$ ,  $p_2$  and  $p_3$  are the positions of the foot landing if we  
 315 run animation steps  $A_1$ ,  $A_2$  and  $A_3$  respectively. The calculated  
 316 barycentric coordinates are then used as weights for the blend-  
 317 ing between animations. A nice property of the barycentric co-  
 318 ordinates is that the sum equals 1, which is a requirement for  
 319 our blending. Finally in order to move the character towards  
 320 the next position, we need to displace the root of the character  
 321 adequately to avoid foot sliding. The final root displacement

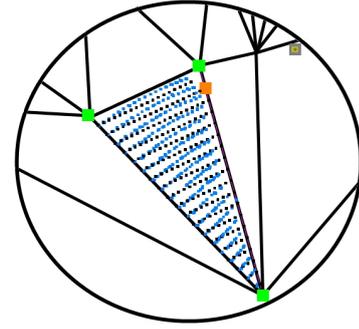


Figure 5: Offsets for different landing positions in a triangle, between barycentric coordinates interpolation (black dots) and blending the whole skeleton using SLERP (blue dots).

322 vector,  $\mathbf{d}'_j$  is calculated as the weighed sum of the root's dis-  
 323 placement of the three selected animation steps (Eq. 2), and  
 324 changes in orientation of the input root trajectory are applied as  
 325 rotations over the ball of the supporting foot.

$$\mathbf{d}'_j = \lambda_1 \cdot \mathbf{d}'_1 + \lambda_2 \cdot \mathbf{d}'_2 + \lambda_3 \cdot \mathbf{d}'_3 \quad (2)$$

326 This provides a final root displacement that is the result of  
 327 interpolating between the three root displacements in order to  
 328 avoid any foot sliding. It is important to notice that the barycen-  
 329 tric coordinates provide the linear interpolation required be-  
 330 tween three points in 2D space to obtain the position  $q_j$ . This  
 331 is an approximation of the real landing position that our char-  
 332 acter will reach, as the result of blending the different poses of  
 333 the three animation clips, using spherical linear interpolation  
 334 (SLERP) with a simple iterative approach as described in [30].

335 Therefore there will be an offset between the desired posi-  
 336 tion  $q_j$  and the position reached after interpolating the three an-  
 337 imations. To illustrate this offset, Fig. 5 shows the points sam-  
 338 pled to compute barycentric coordinates in black, and in blue  
 339 the real landing positions achieved after applying the barycen-  
 340 tric weights to the animation engine and performing blending  
 341 using SLERP. In order to correct this small offset at the same  
 342 time that we adjust the feet to the elevation of the terrain and  
 343 orient the footstep correctly, we incorporate a fast and simple  
 344 IK solver.

#### 345 4.5. Inverse Kinematics

346 An analytical IK solver modifies the leg joints in order to  
 347 reach the desired position at the right time with a pose as close  
 348 as possible to the original motion capture data. For footstep-  
 349 based control, the desired foot position is already encoded in the  
 350 footstep trajectory, and for COM trajectories the final position is  
 351 calculated by projecting the current position of the foot over the  
 352 terrain. The controller feeds the IK system with the end position  
 353 and orientation for each footstep. This allows the system to  
 354 handle footsteps on uneven terrain.

### 355 5. Incorporating Root Movement Fidelity

356 In some scenarios the user may be more interested in fol-  
 357 lowing root velocities than in placing the feet at exact footsteps

358 or with specific walking styles. We present a solution to include  
 359 root movement based interpolation in our current barycentric  
 360 coordinates based interpolator through a user controlled param-  
 361 eter  $\lambda_4$ .

362 For this purpose, we incorporate the locomotion system pre-  
 363 sented by Johansen [9] to produce synthesized motions that  
 364 follow a COM trajectory with correction for uneven terrain.  
 365 During offline analysis, a parametric space is defined using all  
 366 the root velocity vectors extracted from the clips in the motion  
 367 database. For example, a walk forward clip at 1.5 m/s, and a  
 368 left step clip at 0.5 m/s produces a parametric space using the  
 369 root velocity vectors going from the forward direction to the  
 370  $90^\circ$  direction, and with speeds from 0.5 m/s to 1.5 m/s.

371 Given a desired root velocity we define a parametric space  
 372  $\Omega_r$ , and a gradient band interpolator in polar space [9] is created  
 373 to compute the weights for each animation clip to produce the  
 374 final blended result. The gradient band interpolator does not en-  
 375 sure accuracy of the produced parameter values but it does en-  
 376 sure smooth interpolation under dynamically and continuously  
 377 changing parameter values, as with a player-controlled char-  
 378 acter. Once the different clips are blended with the computed  
 379 weights, the system predicts the support foot position at the end  
 380 of the cycle and projects it on the ground to find the exact posi-  
 381 tion where it should land.

382 The root movement based interpolator will select a set of  $k$   
 383 animations  $A_1^r$  to  $A_k^r$  with their corresponding weights:  $w_1, \dots, w_k$ .  
 384 Each of those animations provides a landing position  $p_1^r, \dots, p_k^r$ ,  
 385 and if we only interpolated these animations we would obtain  
 386 the landing point  $r$ .

387 In order to incorporate the output of the polar gradient band  
 388 interpolator in the barycentric coordinates based interpolator  
 389 we proceed as indicated in Algorithm 1.

390 The algorithm first checks whether a vertex of the current  
 391 triangle  $\langle p_1, p_2, p_3 \rangle$  can be replaced by any of the three vertices  
 392 with highest weights selected by the polar band interpolator,  $p_j^r$ ,  
 393  $j \in [1, k]$  (lines 1-13 in the algorithm). This replacement takes  
 394 place if the distance between the two landing positions  $p_i$  and  
 395  $p_j^r$  is within a user input threshold  $\epsilon$  (line 7), and the resulting  
 396 triangle still contains the desired landing position  $q_j$  (function  
 397 *IsInTriangle* returns true if  $q_j$  is inside the new triangle). This  
 398 means that there is another animation that also provides a valid  
 399 triangle and has a root velocity that is closer to the input root  
 400 velocity.

401 Next, function *CalculateRootLanding* computes the landing  
 402 position reached after blending the animations given by the root  
 403 movement interpolator (Eq. 3).

$$r = \sum_{i=1}^k w_i \cdot p_i^r \quad (3)$$

404 Finally, *ComputeWeights* calculates the three  $\lambda_i$  for the next  
 405 footstep  $q_j$  by incorporating a user provided  $\lambda_4$  and the result of  
 406 the polar band interpolator  $r$  (Eq. 4).

$$q_j = \lambda_1 \cdot p_1 + \lambda_2 \cdot p_2 + \lambda_3 \cdot p_3 + \lambda_4 \cdot r \quad (4)$$

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### Algorithm 1 Incorporating root movement fidelity

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#### Input:

- The target position  $q_j$ ,
- The current triangle  $\langle p_1, p_2, p_3 \rangle$ ,
- Root landing positions  $\langle p_1^r, \dots, p_k^r \rangle$ ,
- Animation weights  $\langle w_1, \dots, w_k \rangle | w_1 \geq \dots \geq w_k$ ,
- A user input threshold  $\epsilon$ ,
- A user input weight parameter  $\lambda_4$

#### Output: $\lambda_1, \lambda_2, \lambda_3$

```

1: for  $i = 1$  to 3 do
2:    $u \leftarrow (i + 1) \bmod 3$ 
3:    $v \leftarrow (i + 2) \bmod 3$ 
4:    $j \leftarrow 1$ 
5:    $replaced \leftarrow \text{false}$ 
6:   while  $j \leq 3 \wedge \neg replaced$  do
7:     if  $\|p_i - p_j^r\| \leq \epsilon \wedge \text{IsInTriangle}(q_j, \langle p_j^r, p_u, p_v \rangle)$ 
8:       then
9:          $p_i \leftarrow p_j^r$ 
10:         $replaced \leftarrow \text{true}$ 
11:     end if
12:      $j \leftarrow j + 1$ 
13:   end while
14: end for
15:  $r \leftarrow \text{CalculateRootLanding}(\langle p_1^r, \dots, p_k^r \rangle, \langle w_1, \dots, w_k \rangle)$ 
16:  $\langle \lambda_1, \lambda_2, \lambda_3 \rangle \leftarrow \text{ComputeWeights}(\langle p_1, p_2, p_3 \rangle, \lambda_4, r)$ 

```

---

and  $\lambda_i$  are defined using the following relationship:

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1 \quad (5)$$

407 Since  $w_i$  and  $p_i^r$  are known  $\forall i \in \{1, \dots, k\}$ , and  $\lambda_4$  is a user in-  
 408 put, we have a linear system, where  $\lambda_4$  determines the trade-off  
 409 between following footsteps accurately (if  $\lambda_4 = 0$ ), and simply  
 410 following root movement (if  $\lambda_4 = 1$ ).

411 As the user increases  $\lambda_4$  there will be a value  $\beta \in [0, 1]$  for  
 412 which  $\lambda_1, \lambda_2$  or  $\lambda_3$  will be negative, when solving the system  
 413 of equations formed by eq.4 and eq.5. In order to avoid anima-  
 414 tion artifacts it is necessary to deal only with positive weights,  
 415 therefore we guarantee that the system will only reproduce  $q_j$   
 416 accurately as long as  $\lambda_4 < \beta$ . If we further increase  $\lambda_4$  beyond  
 417 the value  $\beta$  then the algorithm will provide the blending values  
 418 that correspond to a new point  $q'$  which is the result of a linear  
 419 interpolation between  $q_j$  and point  $r$ . When  $\lambda_4 = 1$  the result-  
 420 ing blending will be exclusively the one provided by the root  
 421 movement trajectory since  $\lambda_1 = \lambda_2 = \lambda_3 = 0$ . Fig. 6 illustrates  
 422 this situation.

423 **Time Warping.** Incorporating root velocity in the interpola-  
 424 tion, does not always guarantee that the time constraints as-  
 425 signed per footstep will be satisfied. Therefore once we have  
 426 the final set of animations to interpolate between, with their  
 427 corresponding weights  $\lambda_i, i \in \{1, 2, 3\}$  and  $w_j, j \in [1, k]$ , we  
 428 need to apply time warping. Each input footstep  $f_m$  has a time  
 429 stamp  $\tau_m$  indicating the time at which position  $q_m$  should be  
 430 reached (where  $m \in [1, M]$  and  $M$  is the number of footsteps in  
 431 the input trajectory). The total time of the current motion,  $T$  can

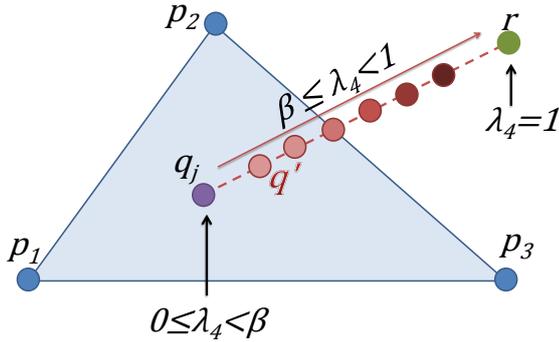


Figure 6: When solving the system of equations given by eq.4 and eq.5, the value of either  $\lambda_1$ ,  $\lambda_2$  or  $\lambda_3$  will be negative when  $\lambda_4 \geq \beta$ . Therefore we need to calculate the barycentric coordinates for a new point  $q'$  which moves linearly from  $q_j$  to  $r$  as the user increases the value of  $\lambda_4$  from  $\beta$  to 1. This means solving the system of equations for  $q'$  instead of  $q_j$ , as it is the closest point to the desired landing position which guarantees that all weights in eq. 5 will be positive.

be calculated as the weighted sum of the time of the animation steps being interpolated:  $T = \sum_{i=1}^3 (\lambda_i \cdot t(A_i)) + \sum_{j=1}^k (w_j \cdot t(A_j))$ . Therefore the time warping factor that needs to be applied can be calculated as:  $warp_m = (\tau_m - \tau_{m-1})/T$ .

**Outside the Convex-Hull.** The footstep parametric space defines a convex-hull delimiting the area where our character can land its feet. When our target footstep position falls inside this area, clips can be interpolated to reach that desired position. But if it falls outside this convex-hull we still want the system to consider and try to reach it. Our solution to handle this problem consists of projecting orthogonally the input landing position  $q$  over the convex-hull to a new position  $q_{proj}$ . Our system then gives the blending weights for  $q_{proj}$  and applies IK to adjust the final position. We include a parameter to define a maximum distance for the IK to set an upper limit on the correction of the landing position. It is important to notice that even if the input trajectory has some footsteps that are unreachable with the current data base of animation clips, our system will provide a synthesized animation that will follow the input trajectory as closely as possible, until it recovers and catches up with future steps in the input trajectory. This situation is similar to the scenarios where the user increases  $\lambda_4$  and then reduces it again.

## 6. Results

The animation system described in the paper is implemented in C# using the Unity 3D Engine [31]. The footstep trajectories used to animate the characters are generated using the method described in [7] or are created by the user. Some difficult scenarios, exercising careful footstep selection, are shown in Fig. 1 and Fig. 7. Agents carefully plant their feet over pillars (Fig. 7-a) or use stepping stones to avoid falling into the water (Fig. 7-b). We show our ability to handle over a hundred agents at 13 FPS (Fig. 7-c and Fig. 9). The supplementary video demon-

strates additional results (high resolution video<sup>2</sup>, low resolution video<sup>3</sup>).

**Obstacle Course.** We exercise the locomotion dexterity of a single animated character in an obstacle course. The character follows a footstep trajectory with different walking gaits, alternating running and walking phases (Fig. 1-a,b), and including sidesteps (Fig. 1-c) and backward motion (Fig. 1-e).

**Stepping Stone Problem.** Stepping stone problems (Fig. 7-b) require careful footstep level precision where constraints require the character to place their feet exactly on top of the stones in order to successfully navigate the environment. Our framework can be coupled with footstep-based controllers to solve these challenging benchmarks.

**Integration with Crowd Simulator.** We integrate our animation system with footstep-based simulators [7]; our character follows the simulated trajectories without compromising its motion fidelity while scaling to handle large crowds of characters (Fig. 7-c).

It is important to mention that the quality of the results depends strongly on the quality of the clips available from the motion capture library. As can be seen in the video, the least precise movements in our results are side steps and back steps. This is due to two reasons: (1) we had a small number of animations compared to other walking gaits, and thus triangles covering that space have larger areas, and (2) interpolation artifacts appear when blending between animations that move in opposite directions (for example a backwards step with a forward step). We believe that having a better and denser sampling in these areas will improve the results. For steps falling in triangles of smaller areas, and with all the vertices in the same quartile we have obtained results of high quality even for difficult animations such as running or performing small jumps.

### 6.1. Foot Placement Accuracy

The presented barycentric coordinates interpolator assumes a small offset between the results of linearly interpolating landing positions from the set of animations being blended, and the actual landing position when calculating spherical linear interpolation over the set of quaternions. This small offset depends on the area of the triangle, so as we incorporate more animations into our data base, we obtain a denser sampling of landing positions and thus reduce both the area of the triangles and the offset. We believe this is a convenient trade off since such a small offset can be eliminated with a simple analytical solver but the efficiency of computing barycentric coordinates offers great performance. It is also important to notice that if exact foot location is not necessary, and the user only needs to indicate small areas for stepping as in the water scenario, then it is not necessary to apply the IK correction. Fig. 8 shows the offset between the landing position and the footstep. The magnitude of the error is illustrated as the height of the red cylinders that are located at the exact location where the foot first strikes.

<sup>2</sup><https://www.dropbox.com/s/o1b9w73qd45fmip/videoCAG.mp4?dl=0>

<sup>3</sup><https://www.dropbox.com/s/sptdz788f2k9ad3g/videoCAGlowRes.mp4?dl=0>

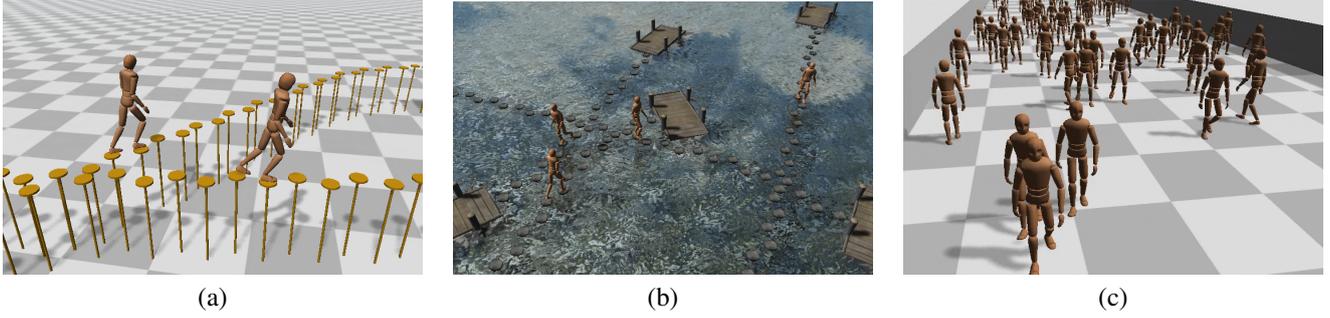


Figure 7: (a) Agents accurately following a footstep trajectory and avoiding falls by carefully stepping over pillars. (b) The stepping stone problem is solved with characters avoiding falls into the water. (c) A crowd of over 100 agents simulated at interactive rates.

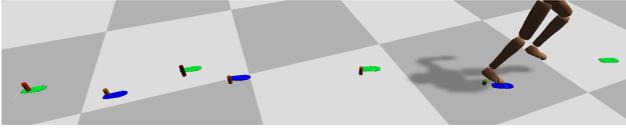


Figure 8: The red columns show the small offset between landing position and the footstep when the IK corrections are not being applied.

## 7. Conclusions and Future Work

We have presented a system that uses multiple parameter spaces to animate fully embodied virtual humans to accurately follow a footstep trajectory respecting root velocities, using a relatively small number of animation clips (24 in our examples). Our method is fast enough to be used with tens of characters in real time (25 FPS) and over a hundred characters at 13 FPS. The method can handle uneven terrain, and can be easily extended to introduce additional locomotion behaviors by grouping new sets of animation clips and generating different parametric spaces. For example, walking and running motions can be blended together, but if we wanted to add crawling motions or jumping motions, it would be better to separate them in different parametric spaces for each style. This will avoid unnatural interpolations that can appear when blending between very different styles. Having different parametric spaces requires some sort of classification, which could initially be done manually but it could also be based on the characteristics of the motion, such as changes in acceleration, maximum heights of the root, length of fly phase, etc. Assuming we can extract the parametric spaces for different animation types, it would also be necessary in some cases to have additional transition clips to switch between very different locomotion types, i.e. crawling and walking.

We do not run physical or biomechanical simulations, and use interpolation and blending between motion capture animations. Our method accuracy depends on the variety of animation clips, while its quality and efficiency depends on the number of clips. A trade-off between efficiency and accuracy is therefore necessary, for which we have found a good equilibrium.

**Limitations.** In order to reduce the dimensionality of the problem, we have not included in our parametric space the orientation of the previous footstep. Ignoring the final orientation of the character at the end of the previous step can induce some discontinuities between footsteps. We mitigate this effect by blending between footsteps automatically for a small amount of time (about 0.2 seconds) at the advantage of reducing the computational time and thus making our method suitable for large groups of agents in real time. Regarding the selection of animation at the end of each footstep, notice that in our database, left and right animation steps are extracted from complete animation cycles that are usually consistent in parameters such as

### 6.2. Performance

Fig. 9 shows the frame rate we obtain as we double the number of agents. It is important to notice that increasing the number of animations would enhance the quality and accuracy of the results, with just a small overhead on the performance.

The average time of the locomotion controller is 0.43ms, this process includes blending animations, IK, the polar band interpolator and our barycentric coordinates based interpolator. The computational cost of our footstep interpolator is 0.2 ms, which is amortized over several frames as the interpolation in  $\Omega_{f_L}$  or  $\Omega_{f_R}$  only need to be performed once per footstep. This time is divided between computing the root movement polar band interpolator which takes 0.155ms and our barycentric coordinates interpolator which takes 0.045ms. Performance results were measured on an Intel Core i7-2600k CPU at 3.40GHz with 16GB RAM.

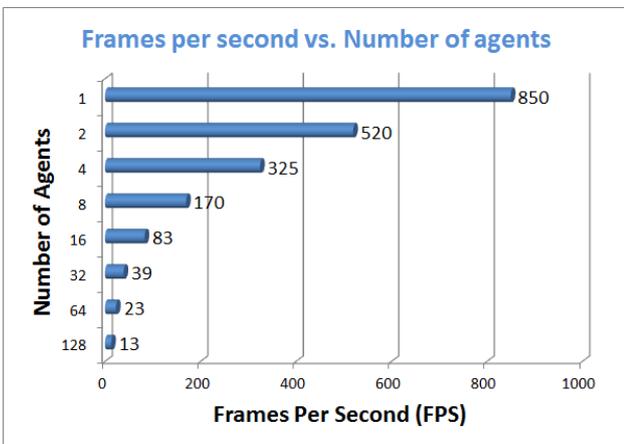


Figure 9: Performance of the Footstep Locomotion System in frames per second as the number of agents increases.

573 velocity, acceleration and walking gait. Therefore for a given  
 574 sequence of steps, the most likely animation steps to be chosen  
 575 will be those extracted from the same set of animation cycles,  
 576 thus resulting in smooth and natural transitions between very  
 577 similar steps. When the characteristics of the steps change drastically,  
 578 then our method needs to blend between steps from very  
 579 different animation cycles. So in general, alternating left/right  
 580 steps results in natural transitions with smooth continuity when  
 581 blending animations, and only when the input step trajectory  
 582 changes drastically between each pair of steps, we may observe  
 583 transitions between animations that feel unnatural. This can  
 584 happen if the step trajectory is done manually with artifacts due  
 585 to the user's lack of experience creating footstep trajectories,  
 586 or for example when the input trajectory forces the character to  
 587 walk over artificially located steps, like crossing a river by stepping  
 588 over stones. We would like to emphasize that this situation  
 589 would also look awkward in the real world and thus the result  
 590 of our synthesized animation may be the desired one.

591 **Future Work.** For future work we would like to extend our  
 592 barycentric coordinates interpolator to 3D space with the third  
 593 coordinate being the root velocity. This will free our system  
 594 from the polar band interpolator which not only takes longer to  
 595 compute but also selects too many animations which results in  
 596 slower blending. One thing to explore could be to interleave the  
 597 execution of the Footstep-based Locomotion Controller from  
 598 different characters in different frames, ensuring we do not execute  
 599 it for all the agents in the crowd.

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