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Highlights

A discrete-time chaotic oscillator based on the logistic map: A secure communication scheme and a simple experiment using Arduino

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- A novel discrete-time chaotic oscillator is conceived using the logistic map.
- Chaos is validated using bifurcation diagram and the corresponding chaotic attractor is shown too.
- A secure communication system is granted including numerical experiments.
- An experiment using the Arduino-UNO board is realized.

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A discrete-time chaotic oscillator based on the logistic map: A secure communication scheme and a simple experiment using Arduino

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Abstract

This paper presents a modified discrete-time chaotic system obtained from the standard logistic map model. Then, a secure communication system is given and numerical experiments are carried out using the conceived discrete-time chaotic oscillator. Moreover, an experiment of our chaotic model is realized using the Arduino-UNO board.

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1. Introduction

The logistic map is considered as a well known non-linear recurrent relation with a single parameter, a_1, given by [1–3]

$$x_1(k + 1) = a_1 x_1(k)(1 - x_1(k)). \tag{1}$$

Basically, the logistic map was postulated to describe the single specie population dynamics of size x_1 being a_1 the growth rate of the population [4]. In biology, this model can approximate the dynamics of many different birth/death processes [4]. In electronic communications and information science fields, the discrete-time logistic map can be used to generate a chaotic signal on chaos-based secure communication system designs [5,6], or as a chaotic noise generator [7],

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¹among others. For instance, the logistic map can be employed to model some processes in chemistry [3], and so on.

³In recent years, new discrete-time chaotic systems, based on the logistic map, have been proposed. For instance, in [3], some modified logistic maps of arbitrary power are studied. Using ⁵delay and q-deformations, some chaotic logistic maps are studied in [8]. The discrete[1] fractional logistic map is analyzed in [9], and the fractional sine and standard maps are examined in [10]. ⁷For an application of a modified logistic map, see [11]. In this paper we will investigate another case of logistic map. Also, we will give a secure communication design.

⁹On the other hand, one of the open-source hardware projects now popular is Arduino. Arduino microcontroller is a single board computer easy to use that has gained considerable attention in ¹¹the hobby and professional market [12–15]. The Arduino hardware is reasonably priced and development software is completely free and easy to use too. To evidence the applicability of our ¹³discrete chaotic oscillator in today's digital technology, we program our discrete chaotic oscillator in Arduino (the Arduino-UNO board).

¹⁵The content of the rest of the paper is as follows. Section 2 gives a modified version of the chaotic logistic map including its bifurcation diagram, its initial conditions sensibility test, and its ¹⁷chaotic attractor. We use the standard logistic map for comparison. Section 3 shows a secure communication scheme along with numerical experiments using our discrete chaotic oscillator. ¹⁹Section 4 presents an experimental realization of the given discrete model using the Arduino-UNO board. Finally, Section 5 states the conclusions.

2. The proposed logistic map

²⁵We grant the following modified version of the logistic map:

$$x_2(k + 1) = -a_2 \, \text{sgn}(x_2(k))(1 - x_2(k)), \tag{2}$$

²⁷where sgn() represents the *signum* function and a_2 is the system parameter. To study its ²⁹dynamics, Fig. 1 shows its bifurcation diagram along with the bifurcation diagram of the standard logistic map. Here after, and according to these bifurcation diagrams, we are going to ³¹use $a_1 = 3.8$ and $a_2 = 1.3$ to have chaotic behaviors on both discrete systems: the standard logistic map and the conceived system, respectively. Thus, Fig. 2 shows the initial conditions sensibility ³³test using initial conditions $x_1(0) = 0.1$ and $x_1(0) = 0.15$, and $x_2(0) = 0.1$ and $x_2(0) = 0.15$, for the standard map, and the modified one, respectively. In red are the discrete-time trajectories ³⁵corresponding to $x_1(0) = 0.1$ and $x_2(0) = 0.1$, and in blue to those corresponding to $x_1(0) = 0.15$ and $x_2(0) = 0.15$. Whereas, Fig. 3 displays their chaotic attractors.

2.1. Lyapunov exponent

⁴¹*Logistic map*: Lyapunov exponent is another mathematical tool to test chaos. Basically, this tool is a quantity that characterizes the rate of separation of infinitesimally close trajectories [2]. ⁴³Let us consider the logistic map (1), on

$$f(x_1(k); a_1) = a_1 x_1(k)(1 - x_1(k)). \tag{3}$$

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[1]In some references, *discrete* systems means *discrete-time* models. So, and depending on the context, these terms can ⁴⁷be used interchangeably.
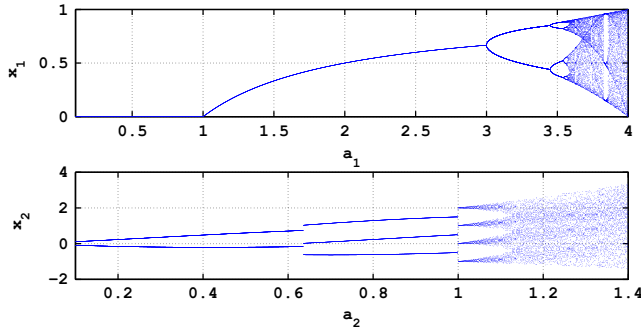
Fig. 1. Bifurcation diagrams (long-term values). Top: for the standard logistic map, Bottom: for the given system.
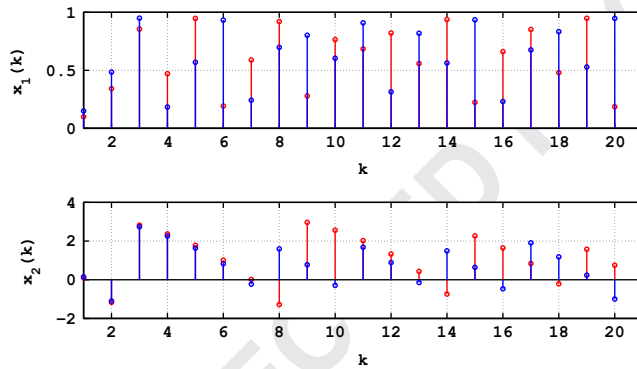

Fig. 2. Initial conditions sensibility test. Top: for the standard logistic map, Bottom: for the given model. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

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For an orbit of $f(x_1(k); a_1)$ starting at $x(0) = x_0$, the Lyapunov exponent, $\lambda(x_1(k), a_1; x_0)$, is

$$\lambda(x_1(k), a_1; x_0) = \lim_{N \to \infty} \frac{1}{N} \sum_{k=0}^{N-1} \ln(|f'(x_1(k); a_1)|), \tag{4}$$

where $f'(x_1(k); a_1) = a_1 - 2a_1 x_1(k)$. By repeating Eq. (4) (but using a large value of N, for instance, $N = 10{,}000$) for different values of parameter a_1, and $x_0 = 0.5$, then the picture shown in Fig. 4 is obtained.

Remark 1. In some numerical experiments of some scientific papers, usually, the initial condition x_0 is slightly different (and generated randomly) when the parameter a_1 is changed in Eq. (4). Qualitatively speaking, the results realized are the same.

The proposed chaotic discrete system: Due to we are dealing with a discontinuous system (because of the signum function term) in our proposed system (2), we can evolve an estimation procedure to obtain the Lyapunov exponent *almost everywhere* (a.e.). To begin with, let us
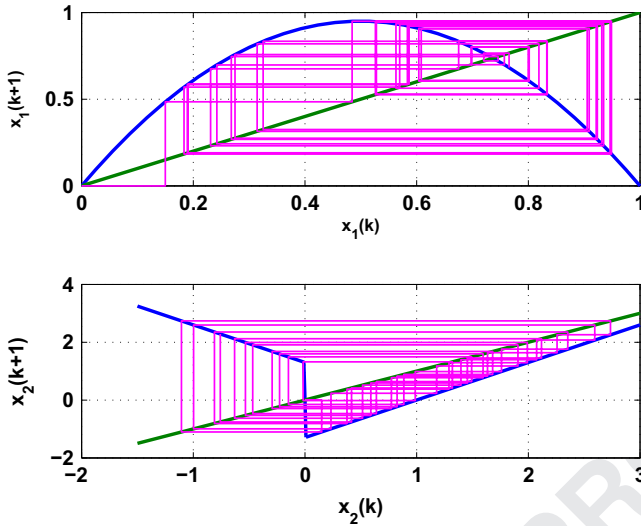
Fig. 3. Chaotic attractors. Top: for the standard logistic map, Bottom: for the proposed modified logistic map.
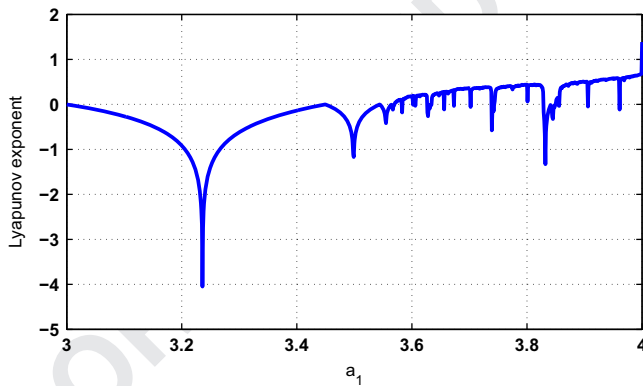

Fig. 4. Lyapunov exponent of the logistic map.

follow the above procedure. In this case, we have

$$f(x_2(k); a_2) = a_2 \, \mathrm{sgn}(x_2(k))(1 - x_2(k)) = \begin{cases} a_2(1 - x_2(k)), & x_2(k) > 0 \\ -a_2(1 - x_2(k)), & x_2(k) < 0. \end{cases} \tag{5}$$

Then,

$$f'(x_2(k); a_2) = \begin{cases} -a_2, & x_2(k) > 0 \\ a_2, & x_2(k) < 0 \end{cases} = -a_2 \, \mathrm{sgn}(x2(k)). \tag{6}$$

On this way, we obtain that $|f'(x_2(k); a_2)| = |-a_2 \, \mathrm{sgn}(x2(k))| = a_2 > 0, \ a.e..$ So, the Lyapunov exponent is positive proving chaos.

3. A secure communication system scheme

To show the applicability of our discrete chaotic model to secure communication system design, we propose the scheme displayed in Fig. 5. In this figure, the discrete chaotic systems, for the transmitter and the receiver, are the system (2). These systems start at the same time with the same initial conditions (we use the value of 0.15). In digital communication systems, and due to we are utilizing discrete-time oscillators, a communication protocol can be designed such that both discrete-time chaotic oscillators start working at the same time using the same initial conditions. This because in fully digital communication systems, the timing information is derived from the samples of the received signal. Then the sampling clock of the receiver with the remote transmit clock is synchronized [16] (this communication stage design is called *carrier synchronization* [17]), facilitating the synchronized data transfer between the transmitter and the receiver.

The zero-order-hold (ZOH) blocks, employed to convert the discrete-time signal to a piece-wise continuous-time signal, are programmed to hold each sample for one second. For instance, Fig. 6 shows the ZOH response for the transmitter system. The low-pass-filter (LPF) corresponding to the receiver system has a transfer function given by

$$G_1(s) = \frac{1}{s+1}, \tag{7}$$

whereas the LPF corresponding to the transmitter system is

$$G_2(s) = \frac{m(t)}{s+m(t)}, \tag{8}$$

where $m(t)$ is the information signal to be encrypted. This information signal is assumed to be a two value one (a binary signal). Fig. 7 shows the numerical experiment results. According to the last figure, the obtained signal $10|e(t)|$ displays some characteristics that can be used to further estimate the transmitted message (see, for instance, [18,19]), using, for example, a filter [20], or some filtering along with a comparator data [21].

Remark 2. From the security point of view, apparently, if new chaotic oscillators are kept in secrecy, the secure of the communication system is increased. On the other hand, the study of resistant to attacks in the proposed system is beyond the scope of this paper and expertise of the author.

4. An experiment using the Arduino-UNO board

There are many Arduino boards. For a complete description of Arduino boards, see [12]. We select the Arduino-UNO. The experiment set-up is shown in Fig. 8. Basically, the Arduino-UNO
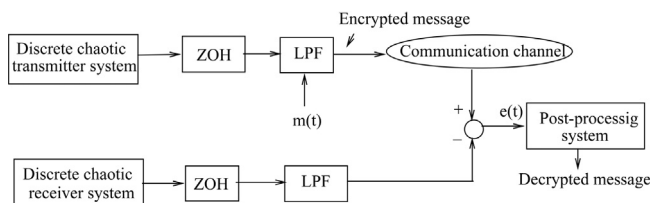

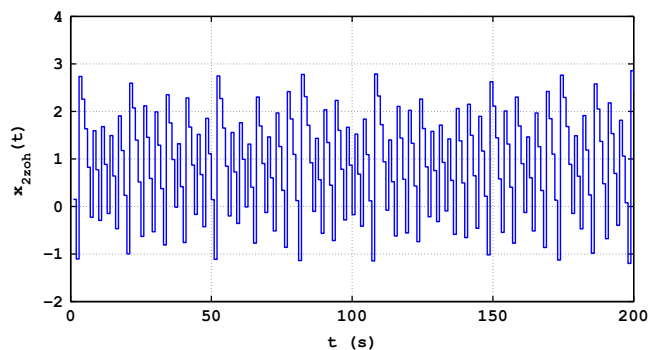Fig. 5. Block diagram of the proposed communication system.

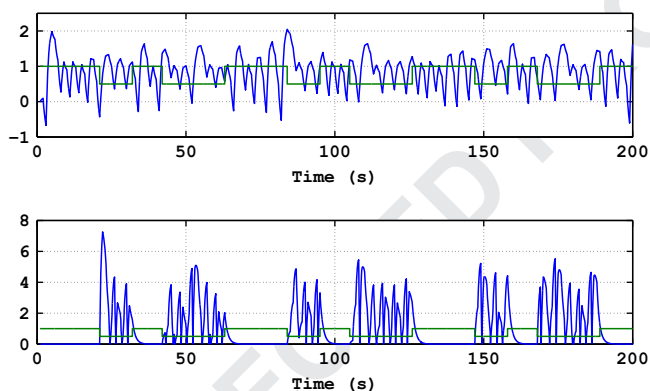Fig. 6. Zero order hold response.

Fig. 7. Numerical experiments of the proposed secure communication design. Top: the green-line is the information signal and the blue-line is the encrypted signal on the communication channel. Bottom: the green-line is the information signal and the blue-signal is $10|e(t)|$. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)
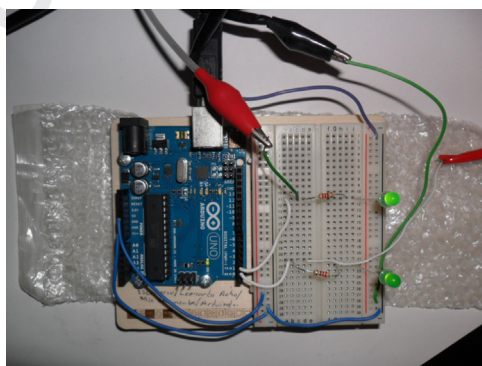

Fig. 8. A photo of the experiment realization using Arduino-UNO.

```
void setup() // one-time actions
{
pinMode(2,OUTPUT); // define pin 2 as an output
pinMode(1,OUTPUT); // define pin 1 as an output
}
void loop() // loop forever
{
double a=1.3, x=0.15;
for (double i=1; i<100000;i++){
x=-a*(1-x)*x/(abs(x)+0.01); // the discrete chaotic oscillator
if (x>1)
{
  digitalWrite(2,HIGH); // activate pin 2
  delay(200);        // for 200 ms
}
else
{
  digitalWrite(2,LOW); //des-activate pin2
  delay(200);        // for 200 ms
}
if (x>0.5)
{
  digitalWrite(1,HIGH); //activate pin 1
  delay(200);        // for 200 ms
}
else
{
  digitalWrite(1,LOW); // des-activate pin 1
  delay(200);        // for 200 ms
}
}
} //END PROGRAM
```

Fig. 9. The program in Arduino-UNO.

pins 1 and 2 are used as digital outputs to turn-on, or turn-off, the corresponding connected Led. Then, the discrete chaotic oscillator (2)[,2] was programmed (see Fig. 9). In this program, the Arduino-UNO output pin 1 is activated when $x_2(k)$ is bigger than 1, otherwise, it is des-activated. The same for the Arduino-UNO output pin 2 but now using the comparison value to 0.5. The

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[2]To implement the signum function, we used the approximation $\text{sgn}(x) \simeq x/(|x| + 0.01)$.
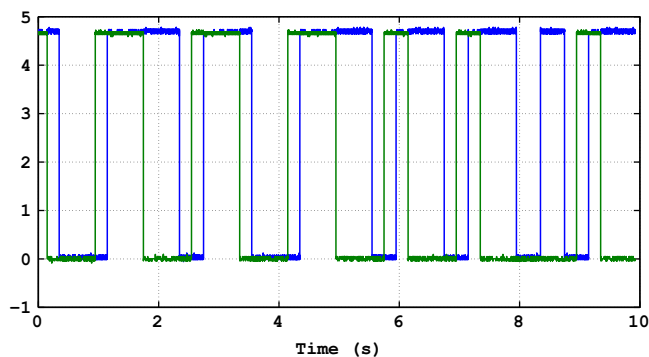
Fig. 10. Experiment results. In blue-line is the Arduino-UNO output at pin 1, and the green-line is the Arduino-UNO output at pin 2. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)
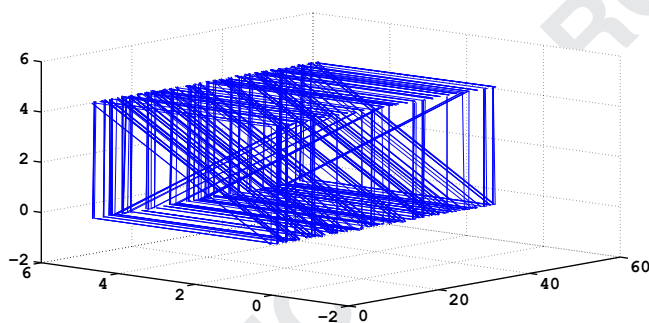

Fig. 11. 3-D experimental result: $(x, y, z) =$ (time (s), Arduino-UNO output at pin 1, Arduino-UNO output at pin 2).

programmed activation and des-activation times were 200 ms. Figs. 10 and 11 display the experiment results.

5. Conclusion

We have presented a novel discrete chaotic system based on the standard logistic map. On the other hand, we offered a secure communication system where numerical experiments were carried out using our discrete chaotic model. Obviously, the best secure system is the one kept in secret (for instance, keeping in secret a new chaotic oscillator). The study of the noisy case on the communication channel is left for a future improved version of the proposed secure communication scheme. Although, in digital communications using fiber-optic links, low-frequency noise can be avoided [22]. Finally, the experiment employing Arduino showed the applicability of using discrete chaotic oscillators in today's digital technology at the hand of almost anybody.

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