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PARTICLE FINITE ELEMENT METHOD APPLIED TO GRANULAR MATERIAL FLOW

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Summary. A numerical model, based on a rate-dependent constitutive model, via a flow formulation, and in the framework of the particle finite element method (PFEM) is proposed. It is settled on the assumption that the powder can be modelled as a continuous medium. The model, provided with the corresponding characterization of the parameters, is able to capture the two fundamental phenomena observed during the granular material flow: 1) the irreversibility of most of the deformation experienced by the material and 2) the energy dissipation of the granular system through the inter-particle friction processes, modelled by the plastic dissipation associated with the material model. Experimental and numerical results have been compared in order to study the viability of the proposed model.

1 INTRODUCTION

During the last twenty years, several research groups have developed or adapt different numerical models to capture the evolution of the most relevant properties during granular flow problems. Most of them have concentrated their efforts on the formulation and implementation in the framework of the DEM. However some limitations can emerge when the method is intended to be used in industrial applications. The first point is the impossibility, for practical reasons, of incorporating to the analysis a number of discrete elements as large as the number of the particles involved in the process. The second point is the computational cost, in which the resolution of the contacts between elements, and the explicit integration of the dynamic equations, via the finite difference method, impose a severe limitation in the time length used for the computation.

Another alternative, in which this work is framed, is the so-called Particle Finite Element Method in which the motion of a representative set of particles is modeled by means of a constitutive flow model.

2 PFEM

The particle finite element method[1] emerged as a natural result of previous explorations in the context of the meshless methods. They can be characterized by the following ingredients: 1) the use of a Lagrangean format for describing the motion. A selected cloud of particles of infinitesimal size (material points) are tracked along the motion to describe the continuum medium properties evolution (position, displacement, velocities, strain, stresses, internal variables etc.). When necessary, the properties of the remaining particles of the continuum medium are obtained by interpolation of the properties at points of that cloud. 2) Numerical computations are done on the basis of a finite element mesh that is constructed at every time step on the basis of the particle positions. Then, Delaunay triangulations, allowing the construction of a finite element mesh for a given sets of nodes, emerge as a suitable meshing procedure. 3) The use of a boundary recognition procedure to identify what particles of the cloud define an external (or internal) boundary. The so-called alpha-shape method constitutes a suitable strategy for this purpose. Small values of the alpha-shape parameter return a boundary constituted of all the particles of the cloud. For a uniformly distributed cloud of particles (with typical separation h) alpha-shape values of $1.1h - 1.5h$ provide a good estimation of the actual boundary.

3 CONSTITUTIVE MODEL FOR THE GRANULAR MATERIAL

The constitutive model formulation is settled on the assumption that the powder can be modelled as a continuous medium. Stresses developed can be related to the deformation rate in the powder, which in turn can be related to the nodal positions and velocity. In the general setting of compressible viscous fluid, the constitutive relation can be written in the form

$$\sigma = pI + 2\mu d \tag{1}$$

in which p is the mean stress, d is the rate of deformation tensor and μ represents the viscosity. In the context of visco-plasticity (Perzyna description) the last expression can be written with some degree of generality as

$$d = \tfrac{1}{\bar{\mu}} \langle F \rangle \frac{\partial G}{\partial \sigma} \tag{2}$$

where $\bar{\mu}$ is a constant 'pseudo-viscosity', $F(\sigma) = 0$ represents the plastic yield condition and G stands for the plastic potential. The angled bracket in (2) represents the Macaulay bracket that takes the value of the argument when positive and is zero otherwise. This term ensures no plastic flow when stresses are below yield

$$\langle F \rangle = F \text{ if } F > 0 \text{ and } \langle F \rangle = 0 \text{ if } F \leq 0 \tag{3}$$

Here, the proposed yield condition is a Drucker Prager yield type surface. The functional form is

$$F_1 = \sqrt{\tfrac{3}{2}} \left\| dev\sigma \right\| + b_1 p - b_2 \tag{4}$$

b_2 is the yield stress under pure shear, here it is reinterpreted as the cohesion of the powder material. The coefficient b_1 is understood as the internal friction coefficient of the continuous flow regimen. The (non-associated) flow rule is defined from the plastic potential

$$G(\sigma) = \|\operatorname{dev}\sigma\| \tag{5}$$

which defines a deviatoric strain rate. For this case the vector flow can be written as

$$\left(\frac{\partial G}{\partial \sigma}\right) = \frac{\operatorname{dev}\sigma}{\|\operatorname{dev}\sigma\|} \tag{6}$$

When flow occurs, $F > 0$ and then using (4) and (6) we can obtain the viscosity μ as the solution of the following expression

$$2\mu = \bar{\mu} + \frac{b_2 - b_1 p}{\|\operatorname{dev} d\|} \tag{7}$$

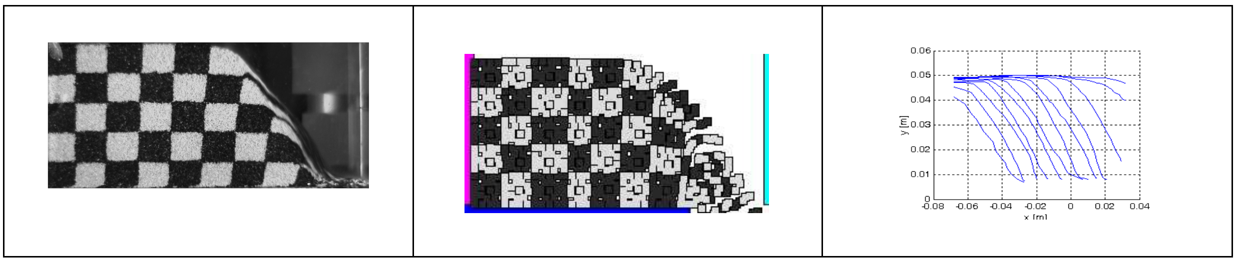
Using both the constitutive relation (1) and the viscosity (7), an expression for the deviatoric part of the stresses, in terms of the strain rate, can be fully obtained.
In the case of fluids, or in general for incompressible materials, the mean stress is obtained from of the incompressibility condition. Here a compressibility law is adopted and the mean stress is expressed as

$$\dot{p} = \kappa \dot{\varepsilon}_v \tag{8}$$

where $\dot{\varepsilon}_v = tr\, d$ is the volumetric strain rate and κ is understood as the bulk modulus of the powder.

4 NUMERICAL RESULTS

First example focuses on the material characterization of the internal friction parameter of the constitutive model. The study focused on the behavior of the powder while it is delivered into the die. Details of the experiment can be found in [3]. Figure 1 shows a typical flow pattern during the filling; experimental results are grouped on the left, numerical ones are on the center and the profiles evolution is on the right. Second example shows the evolution position of the granular flow when the material, by the action of the gravity, moves from the silo, goes down the inclined shute an fill the deposit. Two different internal friction were used; $k = 0.1$ and $k = 0.8$.



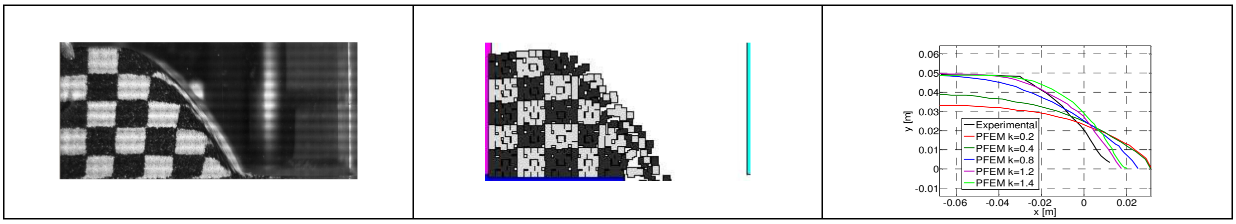


Figure. 1. Comparisons between the movements recorded by a high video system and the numerical results obtained using PFEM

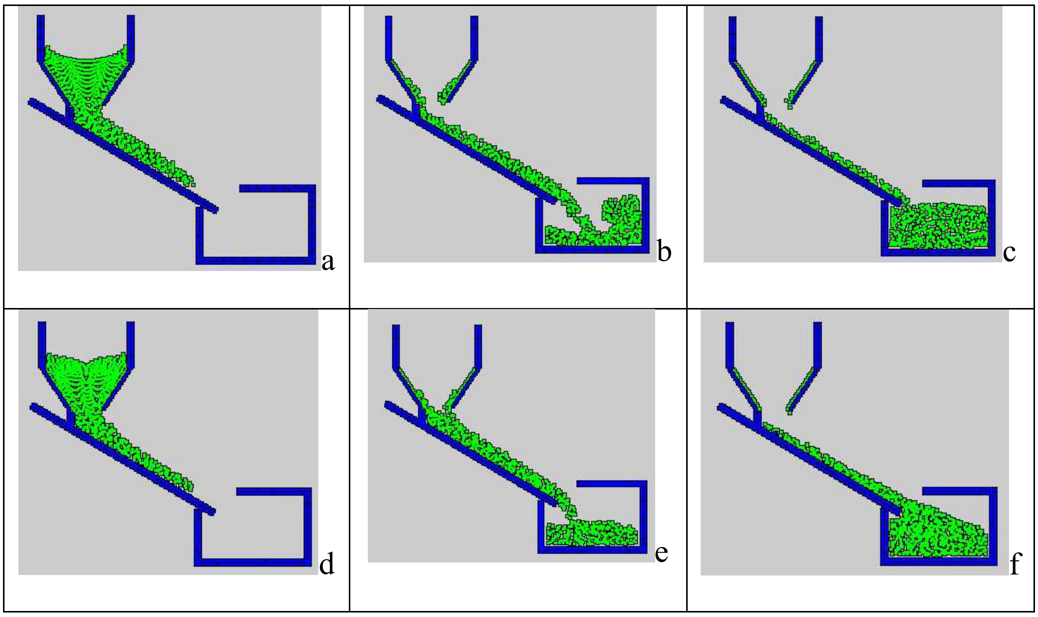


Figure. 2. Particle position evolution for two internal friction angles. For $k = 0.1$ figures a,b, and c. and for $k = 0.8$ figures d, e and f

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