

CONFINEMENT OF SPACECRAFT SWARMS IN J2-PERTURBED ORBITS ABOUT THE EARTH:draft

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ABSTRACT

I INTRODUCTION

The concept of formation flying has been growing in interest over the past few years, due to the advantages that gives over a single (and bigger) spacecraft. However, the technical issues that have to be solved to obtain the tight constraints that a formation has to achieve have provoked a delay in the implementation of this technology. This fact, added to the appearance of very small satellites, has induced to the apparition of a new concept: swarms of spacecraft. Swarms consist of a large number of spacecraft that flight together but, differently from formation flying, they do not have to maintain a tight distance between them.

Swarms of spacecraft were introduced [4] due to the robustness that give a lot of spacecraft, that do not depend on the failure of a single one, and their ability to cover a large area with small (and cheaper) spacecraft.

The main problem that has to be solved with the swarms of spacecraft is to find trajectories for each of the spacecraft that prevent collision between the swarms and that minimize the fuel consumption of each of the spacecraft. Note that the swarms are planned to have very small spacecraft, that can take only a few propellant so fuel consumption is a critical parameter. But the issues that have to be solved with a swarm are different from formation flying: there are lots of possible collisions between spacecraft, and the methodologies that worked with a few spacecraft now have to be modified or changed, since the number of spacecraft in the swarm is two orders of magnitude bigger than the one in a formation.

There have been studies of the reliability of a swarm [3], based on nano-satellites, that give a study for the number of satellites that can be working as a period of time, with good results for a 5-10 anys time step.

The strategies to control the swarm are based in three main concepts. The first one studies the global movement of the swarm as the ones that we can find in nature [5], where each of the spacecraft has behavior depending on the position and velocity of closest neighbors. The second one are the strategies that include the provision of collision-free orbits for long periods of time[1]. Finally, there is the strategy to keep the relative distance between spacecraft [2].

The main contribution of this paper is the adaptation of a methodology proposed for reconfigurations of spacecraft [6] to swarms, taking into account that the higher complexity of the problem needs some adjustments. These adjustments are based on the strategies of swarms in nature (there are only reconfigurations based on the neighbour states). The statement of the problem is to calculate the cost to maintain the spacecraft of the swarm for a year in a sphere of radius R for a year, avoiding collisions and minimizing the fuel usage.

II METHODOLOGY

We consider the swarm as a formation of N spacecraft (being N a big number). Each spacecraft of the swarm is located in the vicinity of a orbit about the Earth, at a maximum distance R of this orbit. On each revolution of the orbit about the Earth, we apply a control to some of

the spacecraft with the objective that all the spacecraft are kept at a sphere of radius R about the orbit and that there is no collision between the spacecraft.

This control, and the cost for each spacecraft, is computed using a variational numerical methodology based on finite elements, that is fully presented in [?, 6]. We consider a J2-perturbed orbit about the Earth, that is the base orbit for the swarm. Then, we consider a coordinate system that is centered in this orbit. As the spacecraft are maintained at a maximum distance R from the orbit, being R a few hundreds of meters, we take the equations of motion of each spacecraft of the swarm as the linearized equations about this orbit. Given this equations, we add a control $\mathbf{U}_i(t)$, that is of the form $(0, 0, 0, u_i^x(t), u_i^y(t), u_i^z(t))^t$. The initial states are fixed, but the final state is free, as long as the position of all of the spacecraft are inside the sphere. So, the equations that have to be solved are

$$\begin{cases} \dot{\mathbf{X}}_i(t) = \mathbf{A}(t)\mathbf{X}_i(t) + \mathbf{U}_i(t) \\ \mathbf{X}_i(0) = \mathbf{X}_i^0 \end{cases} \quad (1)$$

where \mathbf{X}_i^0 stands for the initial state of the i -th spacecraft of the formation.

The goal is to find optimal controls, $\mathbf{U}_1, \dots, \mathbf{U}_N$, subjected to the problem constraints, which are collision avoidance and that $\mathbf{X}_i(t) \leq R$ in all the time interval.

The control $\mathbf{U}(t)$ is obtained with the methodology that is based on the finite element method. The period of the base orbit, T , is divided in a mesh of M elements, which are subintervals of the time domain, $[0, T]$. The nodes of the mesh are the borders of the elements, so each element has two nodes, that are shared with the neighboring elements. The controls are in a form of delta-v that are applied to the time corresponding to the nodes.

The initial problem is then reduced to an optimization problem using the finite element theory. The variables of the problem are related to the states of each spacecraft (the number of variables is then $6MN$), the constraints are collision avoidance and the maintainance in the sphere and the optimization function is related to the fuel expenditure.

Both collision avoidance and maintainance in the sphere check that the conditions are fulfilled on each element of the mesh. Collision avoidance checks, for each pair of spacecraft, and for each element, that the distance between the spacecraft in the element is greater than a security distance r . This gives $MN(N-1)/2$ constraints. Maintainance in the sphere checks that the spacecraft is in the sphere for each element, that is MN constraints. In total, the number of constraints is $MN(N+1)/2$.

Note that the cost of the reconfiguration depends on the mesh used to compute the cost. The mesh is computed via a remeshing strategy [?], that tends to obtain a bang-bang

solution when there are no collision risks and obtains low-thrust based solutions with a maximum error when there are collision risks.

II.1 Strategies to reduce the computation time

As it has been stated, at each revolution, the idea is to maintain the spacecraft in a given sphere of radius R , and avoid collision between spacecraft. For a swarm of 50 spacecraft, and with a mesh of 50 elements, this means that the optimization problem would have 15000 variables and 63750 constraints, which is not solvable in a reasonable time (note that this is only for a revolution of the swarm about the Earth, and the objective is to compute the cost for a year).

The main strategy to reduce the computation time is to take into account that each spacecraft is only influenced by the ones that are nearby. This means that if a single spacecraft has no collision risk with others and it does not exit the sphere in a revolution, it can be let out of the optimizer, simplifying the problem. So, on each step (a revolution about the Earth) we check which are the spacecraft that are more likely to have an issue (collide with a spacecraft, or go outside the sphere) and these are the spacecraft that enter in the optimization problem.

Additionally to this fact, and due to the higher cost of solving optimization problems with a high number of variables, some of the spacecraft can be included in the optimization algorithm in a preventive way: these are the spacecraft that are near the edge of the sphere and the spacecraft that have a possible collision risk in the following periods.

Specifically, the spacecraft that enter the optimizer are:

- For the maintenance about the Earth: all the spacecraft that exit the sphere of radius R about the base orbit enter to the optimizer. If the number of spacecraft that enter the optimizer in this step is n , less than three, then a maximum of $3 - n$ spacecraft are put in the optimizer in a preventive way. These are the $3 - n$ outermost spacecraft, as long as they are outside the sphere of radius $3R/4$ at the end of the period. These spacecraft enter into the optimizer with only one constraint for each of them: their position must be in the $3R/4$ -radius sphere at the end of the period.
- For the collision avoidance: when there is a collision between a pair (or more) spacecraft, one of them has to enter to the optimizer in order to avoid the collision. We note here that the end of a spacecraft lifetime is when the spacecraft ends its fuel. Therefore,

to give the swarm a bigger lifetime, the fuel consumption should be distributed equally in all spacecraft. For this purpose, the methodology penalizes the spacecraft with less accumulated fuel consumption: the spacecraft that has the biggest amount of accumulated consumption does not modify its trajectory in that revolution, while the other spacecraft that collide with it do. This is solved by entering the spacecraft with lower cost in the optimizer as a spacecraft to modify the trajectory, while the other spacecraft only enters the optimizer as a set of collision constraints. Again, in order to avoid a large number of collision avoidance spacecraft in a single revolution about the Earth, if there are less than 3 spacecraft that enter the optimization problem, we add a number of spacecraft in order to complete a set of three spacecraft, that are at the end at a distance greater than $3r$.

In summary, the problem, in most cases, consists on a $36M$ variable problem, with at most $9M$ constraints, which is possible to solve in a reasonable time.

Of course, there could be collisions within one of the spacecraft that was in the optimizer and some spacecraft that were not taking into account for the collision avoidance. After the optimization problem, we check that there is no collision. Otherways, we do again the same process, including the additional constraints necessary in order to avoid collisions.

II.2 Parameters considered in the calculus of the cost

The objective of this paper is to make an study of the cost of maintaining a swarm about the Earth. The cost of maintaining the swarm depends on a couple of factors, some of them inherent to the configuration of the swarm and others depend on the base orbit.

The parameters inherent to the swarm are the ones that are given by the nature of the spacecraft:

- The number of spacecraft in the swarm, N .
- The maximum distance to the base orbit R : This parameter is given by the maximum distance that can be achieved between each pair of spacecraft of the swarm.
- The security distance between spacecraft r , which is given by the size of the spacecraft.
- The initial configuration of the spacecraft in the swarm.

For the purposes of this paper, the initial configuration of the spacecraft has been chosed randomly, but fulfilling two conditions: the first one, is that the initial configuration satisfies the constraints of the problem in an strict way (the states of the spacecraft are chosen randomly in the sphere of radius $3R/4$, and with a minimum distance between spacecraft of $3r$, which assures that there are no feasibility problems in the solution in the first iterations), and the second one is that the swarm is centered at the orbit (meaning that the mean of the states of all the spacecraft is the same as the orbit).

In the case of the other parameters, it is obvious that the cost of maintenance grows with the number of spacecraft and the security distance and decreases R . We have centered the study in a combination of the three parameters, for instance, which can be the radius of the sphere containing the swarm, R , for a given number of spacecraft and a given security distance.

The orbital parameters considered are the semi-major axis, the inclination and the excentricity of the base orbit.

III RESULTS

The results are divided in three different parts: the first one fixes an orbit about the Earth, and studies how the parameters inherent to the swarm influence the total cost; the second one fixes a swarm (N , R and r , but it also fixes the initial states for the spacecraft) and studies the influence of the orbital parameters; and finally, in the last one, studies which is the evolution of some of the parameters of the swarm (the minimum distance between spacecraft on each revolution, the number of spacecraft that enter the optimizer, the maximum distance from the spacecraft to the base orbit).

III.1 Influence of the swarm parameters

The objective of this section is to compute the influence of the swarm parameters, independently of the orbit. Therefore, the orbit is fixed for all the computations. For the following results, the orbit is Molniya-type orbit, integrated with the J2-perturbed model about the Earth, with a semi-major axis of 26500 km, a excentricity of 0.72 and a inclination of 63.4 degrees. The swarm is maintained on a sphere of radius R about the orbit for 733 revolutions, which is roughly a year.

The parameters of the swarm are changed in the following ranges:

- The number of spacecraft of the swarm is from 50 to 100.
- The minimum security distance between spacecraft is 10 meters and the maximum is 20 meters.

- The radius of the confinement sphere is in the range [200, 1000] meters.

For a formation, it is important to know the maximum number of spacecraft that can be maintained, and how the cost is growing with the number of spacecraft. Additionally, when considering the cost of maintaining the swarm, there are two important parameters that gives us the lifetime of the swarm: one of them is the maximum fuel consumption for a spacecraft (that gives us the time when the spacecraft is no longer operational), and the mean cost consumption (that gives us an idea of the lifetime of the swarm). Given the base orbit, we have studied the cost of maintaining a variable number of spacecraft for a year in that orbit, with a security distance of 20 meters and within a range of 200 mters. The results, showed in the left side of figure 1, show that both the mean and the maximum cost grow in a linear way. The reason is that there is plenty of space in the sphere for the spacecraft. In the right side of the figure, we show how, when reducing R to only 100 meters, the cost grows in an exponentially way.

IV CONCLUSIONS

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REFERENCES

- [1] Morgan, D. and Chung, S. *Swarm-Keeping Strategies for Spacecraft Under J2 and Atmospheric Drag Perturbations*. Journal of Guidance, Control, and Dynamics, Vol. 35, No. 5, pp. 1492–1506. doi: 10.2514/1.55705, 2012.
- [2] Mazal, L., Mingotti, G. and Gurfil, P. *Optimal OnOff Cooperative Maneuvers for Long-Term Satellite Cluster Flight*. Journal of Guidance, Control, and Dynamics, Vol. 37, No. 2, pp. 391–402, 2014.
- [3] Engelen, S., Gill, E. and Verhoeven, C. *On the reliability, availability, and throughput of satellite swarms*. Aerospace and Electronic Systems, IEEE Transactions on , vol.50, no.2, pp.1027–1037, 2014.
- [4] Verhoeven, C., Bentum, M., Monna, B., Rotteveel, J. and Guo, J. *On the origin of satellite swarms*. Acta Astronautica, 68, 7–8, pp. 1392–1395, 2011.
- [5] Sabatini, M. and Palmerini, G. *Collective control of spacecraft swarms for space exploration*. Celestial Mechanics and Dynamical Astronomy, Volume 105, Issue 1–3, pp 229–244, 2009.
- [6] Garcia-Taberner, L. *Proximity Maneuvering of Libration Point Orbit Formations Using Adapted Finite Element Methods*. PhD Dissertation. Universitat Politècnica de Catalunya. Barcelona, 2010.
- [7] Garcia-Taberner, L, Masdemont, J. J. *FEFF Methodology for Spacecraft Formations Reconfiguration in the Vicinity of Libration Points*. Acta Astronautica, Vol. 67, Issues 7-8, 810–817, doi:10.1016, 2010.

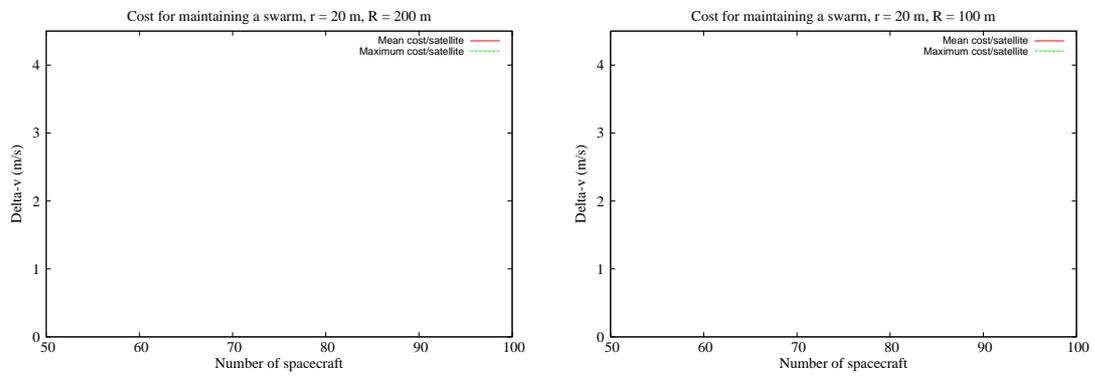


Figure 1: Cost of maintaining a swarm for a year, with fixed security distance and maximum distance to the orbit. The cost for satellite is growing in a linear way when there is enough space, but when there is no much space the cost is growing in an exponential way.