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THE ROLE OF SYMBOLIC LANGUAGE IN THE TRANSFORMATION OF MATHEMATICS

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ABSTRACT

One important factor to be considered in the process of algebrization of mathematics is the emergence of symbolic language in the seventeenth century. Focussing on three works, *In Artem analyticen Isagoge* (1591) by François Viète (1540-1603); *Cursus Mathematicus* (1634-1637-1642) by Pierre Hérigone (1580-1643) and *Geometriae Speciosae Elementa* (1659) by Pietro Mengoli (1626/7-1686), in this article we analyse two relevant aspects of symbolic language: the significance of the notation in the symbolic language and the role of Hérigone's new symbolic method. This analysis allows us to better understand the role played by this circulation of ideas in the formative process of symbolic language in mathematics.

1. Introduction

The creation of a formal language was fundamental to the process of making algebra part of mathematics. One important factor to be considered in this process is the beginning of specious language as a new language for mathematics in the seventeenth century. The use of this new symbolic language was sometimes considered by the authors as an art or as a procedure for expressing ideas that already existed.Some authors, like those under study, believed that this symbolic language was useful for clarifying the understanding of mathematical ideas and also for finding new mathematical results.

According to this perception of a symbolic language, we analyze some relations between the following three works, referred to in chronological order: *In Artem analyticen Isagoge* (1591) by François Viète (1540-1603); *Cursus Mathematicus* (1634-1637-1642) by Pierre Hérigone (1580-1643) and *Geometriae Speciosae Elementa* (1659) by Pietro Mengoli (1626/7-1686) .

The publication in 1591 of *In artem analyticen isagoge* by Viète constituted an important step forward in the development of a symbolic language for mathematics. Viète introduced the specious logistic, therefore the symbols of his analytic art (or algebra) can be used to represent not just numbers but also values of any abstract magnitude. In addition, he used separate letters to represent both the known and the unknown quantities, and was thus able to investigate equations in a general form. Using this symbolic language, Viète demonstrated the usefulness of algebraic procedures for analyzing and solving problems in arithmetic, geometry and trigonometry.[1] As his work came to prominence at the beginning of the 17th century, other authors also began to consider the utility of symbolic language and of algebraic procedures for solving all kinds of problems.

Viète's work was circulated through various texts on algebra, such as the *Algebra* section of Hérigone's *Cursus Mathematicus* in 1634.[2] In fact, Hérigone wrote an encyclopaedic textbook of pure and mixed mathematics consisting of five volumes (six volumes in the second

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[1] Viète published several works for showing the usefulness of this analytic art. On Viète's works see: Viète (1970) and Giusti (1992).

[2] Hérigone's algebra consists of 20 chapters and includes: 1: Several definitions and notations. 2, 3: Operations involving simple and compound algebraic expressions. 4: Operations involving ratios. 5: Proofs of several theorems. 6, 7: Rules for dealing with equations, which are the same as those in Viète's work [These rules were: the reduction of fractions to the same denominator ("isomerie"), the reduction of the coefficient of the highest degree ("parabolisme"), the depression of the degree ("hypobibasme") and the transposition of terms ("antithese")]. 8: An examination of theorems by "poristics". 9: Rules of the "rhetique" or exegetic in equations up to the second degree. 10–13: Solutions of several problems and geometric questions using proofs (determined by means of analysis). 14: Solutions of several "ambiguous" equations. 15: Solutions of problems concerning squares and cubes, referred to as Diophantus' problems. 16–19: Calculation of irrational numbers. 20: Several solutions of "affected" (negative sign) powers.

edition) entitled in full *Cursus Mathematicus, nova, brevi et clara methodo demonstratus, per notas reales & universales, Citra usum cuiuscunque idiomatis, intellectu faciles*[3]*.* Published in parallel French and Latin columns arranged on the same page, the first four volumes appeared in 1634, the fifth in 1637, and a sixth in 1642 as a supplement to the second edition. Hérigone's stated aim in the *Cursus* was to introduce a symbolic language as a universal language for dealing with both pure and mixed mathematics by means of an easier and briefer new method.[4]

Hérigone's *Cursus* reached Italy by way of Santini, Galileo and Cavalieri,[5] and it was there that it was most influential.[6] It was used in

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[3] The first and second volumes of *Cursus* deal with pure mathematics. The first volume contains Euclid's *Elements* and *Data*, and Apollonius's *Coniques*. The second volume is devoted to arithmetic and algebra. The third and fourth volumes deal with mixed mathematics, that is to say, with the mathematics required for practical geometry, military or mechanical uses, geography, and navigation. The fifth and last volume of the first edition, published in 1637, includes spherical trigonometry and music. Later, in the second edition (1642), he adds the sixth and final volume, which contains two parts dealing with algebra; it also deals with perspective and astronomy.

[4] Indeed Gino Loria has already signaled this idea in 1894, see Loria (1894, 110-112).

[5] Cifoletti (1990, 158) states that Antonio Santini explained to Galileo in a letter dated 21 September 1641 that he had sent him Hérigone's *Cursus.* Galileo then sent it to Cavalieri.

[6] On the influence of Hérigone's *Cursus,* see Massa (2008, 298-299).

particular by Mengoli in his *Geometriae Speciosae Elementa* (1659).[7] It is a text in pure mathematics consisting of 472 pages with six *Elementa* whose title: "Elements of Specious Geometry" already indicates the

[7] *Geometriae Speciosae Elementa* (1659) has an introduction entitled *Lectori elementario*, which provides an overview of the six *Elementa*, or individually titled chapters, that follow. In the first *Elementum*, *De potestatibus, à radice binomia, et residua* (pp. 1-19), Mengoli gives the first 10 powers of a binomial given with letters for both addition and subtraction, and says that it is possible to extend his result to higher powers. The second, *De innumerabilibus numerosis progressionibus* (pp. 20-94), contains calculations of numerous summations of powers and products of powers in Mengoli's own notation, as well as demonstrations of certain identities. In the third, *De quasi proportionibus* (pp. 95-147), he defines the ratios "quasi zero", "quasi infinity", "quasi equality" and "quasi a number". With these definitions, he constructs a theory of quasi proportions on the basis of the theory of proportions found in the fifth book of Euclid's *Elements*. The fourth *Elementum*, *De rationibus logarithmicis* (pp. 148-200), provides a complete theory of logarithmical proportions. He constructed a theory of proportions between the ratios in the same manner as Euclid did with magnitudes in the fifth book of *Elements*. From this new theory in the fifth *Elementum*, *De propriis rationum logarithmis* (pp. 201-347) he found a method for calculation of the logarithm of a ratio and deduced many useful properties of the ratios and their powers. Finally, the sixth *Elementum, De innumerabilibus quadraturis* (pp. 348-392) calculates the quadratures of curves determined by algebraic expressions now represented by $y = K.\ x^m.\ (t-x)^n$. A detailed analysis of this work can be found in Massa (2006).

singular use of symbolic language in this work, and particularly in geometry. Mengoli unintentionally created a new field, a "specious geometry", modelled on Viète's "specious algebra" through Hérigone's influence, since he worked with "specious" language, that is to say, symbols used to represent not just numbers, but also values of any abstract magnitudes. Mengoli acknowledges Viète's and Hérigone's influences at the beginning of the book. In the introductory letter to Fernando Riario, Mengoli reveals his sources in a reference to Viète's algebra and he also claims as a source Hérigone's algebra: "To those symbols that Viète, Hérigone, Beaugrand (...)".[8] Actually Mengoli uses Hérigone's new symbolic method to deal with limits, logarithms and quadratures[9].

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[8] Quibus characteribus à Vietta, Herigonio, Beaugrand...(Mengoli, 1659, 12).

[9] Mengoli, who was influenced by Hérigone's idea of symbolic language as a powerful tool, introduces symbolic language into the theory of proportions from Euclid's *Elements*. He extends this theory and creates two new theories: the theory of quasi proportions and the theory of logarithmic proportions (Massa, 1997, 257–280; Massa, 2003, 457–474). Mengoli hardly uses geometric representations at all in his works. He works directly with algebraic expressions of geometric figure. On Mengoli's figures and its quadratures see Massa (2006) and Massa-Delshams (2009).

Since in our previous works we have shown some evidence of the relation between these authors,[10] we may now pose some questions: In what sense can we speak about reception or appropriation of knowledge between these authors? Referring to the new symbolic method introduced by Hérigone, one may ask: How was symbolic language used and understood by Hérigone? What did Hérigone's new symbolic method contribute to the understanding, teaching and validation of mathematical knowledge? Thus, the aim of this article is to analyse two relevant aspects of symbolic language in the relationships between these three works: the significance of the notation in the symbolic language and the role of Hérigone's new symbolic method. This analysis allows us to better understand the role played by this circulation of ideas in the formative process of symbolic language in mathematics.

[10] In our previous work we have shown that Hérigone in "Algebra", section of volume 2, presents the same parts as Viète's works and generally used Viète's statements. However, Hérigone's notation, presentation and procedures in his algebraic proofs were very different from Viète's. On a comparative analysis between Viète's specious algebra and Hérigone's algebra, see Massa (2008).

2. On notation: from Viète's indeterminate to Mengoli's determinable indeterminate quantity

The language used in mathematics before the seventeenth century was mainly rhetorical and then later rhetorical with abbreviations.[11] For instance, in his treatise *Al-kitab almukhtasar fi hisâb al-jabr wa'l-muqabala* (c. 825), Al-Khwarizmi (780-850) describes different kinds of equations using rhetorical explanations. His proofs are given in the form of codified statements. There are no symbols in his work. Later, when Leonardo de Pisa (1170-1240) (known as Fibonacci) expresses the Arabic rules in his *Liber Abaci* (1202), he uses "radix" to represent the "thing" or unknown quantity (also called "res" by other authors) and the word "census" or "ce" to represent the square power. This rhetorical language with some abbreviations continued to be used in several algebraic works in the early Italian Renaissance, such as *Summa de Arithmetica, Geometria, Proportioni e Proportionalità* (1494) by Luca Pacioli

[11] On the different expressions of notations, see Cajori (1928-29).

(1445-1514),[12] and later in *Ars Magna Sive de Regulis Algebraicis* (1545) by Girolamo Cardano (1501-1576). To represent unknown quantities, the first power named "cosa" was abbreviated to "co.", the square or "census" to "ce.", the cube to "cu.", etc.. The influence of German algebras, nowadays named "cossic" algebras,[13] particularly texts such as *Die Coss* (1525) by Christoph Rudolff (1499-1545), and over all *Arithmetica Integra* (1543) by Michael Stifel (1487-1567) was also significant. In German algebras for representing the powers of unknown quantities they generally used a different symbol for each power.[14] In the sixteenth century we can also quote Marco Aurel's work that was one of the first treatises containing algebra to appear in print on the Iberian Peninsula, *Libro primero de Arithmetica Algebratica* (Valencia, 1552), also the

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[12] Høyrup provided an account of the innovations in Italian abacus algebra and referred to mid-fourteenth-century formal calculations of fractions. See Høyrup (2010).

[13] This name derives from the treatment of problems with an unknown quantity called "cosa".

[14] In France, we can quote the *Tryparty* by Nicolas Chuquet and over all the printed works of de la Roche (Heeffer, 2010). Also Jacques Peletier (1517-1582) with *L'Algèbre* (Lyon, 1554), Jean Borrel (Johannes Buteo, 1492-1572) that wrote *Logistica Quae et Arithmetica Vulgo Dicitur* (Lyon, 1559) or Pierre de la Ramée (Petrus Ramus, 1515-1572) that wrote *Algebra* (Paris, 1560) using his own symbolism and terminology. Consequently, there was no clear algebraist's line in France, but rather many individual contributions (Van Egmond, 1988, 141; Cifoletti, 1990, 121-142).

publication of the *Compendio de la Regla de la Cosa o Arte Mayor* (Burgos, 1558) by Juan Pérez de Moya and six years later, the *Arithmética* (Barcelona, 1564) by Antic Roca, all three with different notation but with the same significance of powers in a continued proportion, provide a solid foundation of "Spanish Arte Mayor" as showed by Massa (2012).

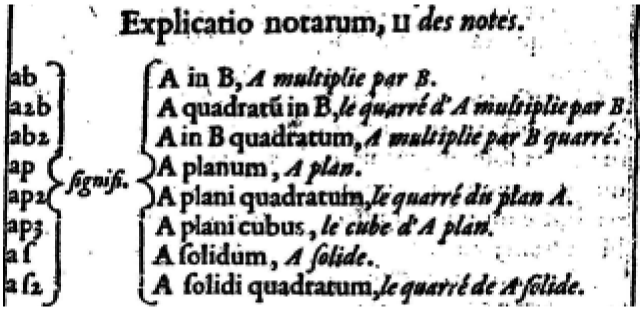
During the seventeenth century the notation and formalism of algebraic expressions evolved after the works by Viète had been published. However, there were no unifying criteria and so for many years different notations were used in algebraic works.[15]

In order to address the circulation and influence of symbolic language of the works under study, one should therefore consider the notation first. If one observes only the notation used by these three authors one may be led to believe that there is no relation between them. There are in fact only a few coincidences between Hérigone's and Mengoli's notation. See table below:

[15] For instance, in the seventeenth century, if we consider the symbolic language in *Artis Analyticae Praxis ad Aequationes Algebraicas Resolvendas* (1631) by Thomas Harriot (c.1560-1621) and in *Clavis Mathematica* (1631) by William Oughtred (1573-1660), who publicized Viète's work in England, we realize that their notations are quite different (Stedall, 2002, 55-125).

Signs	**Viète (1591)**	**Hérigone (1634)**	**Mengoli (1659)**
Equality Greater than Less than	aequalis	2/2 3/2 2/3	: Maior quam Minor quam
Product of a and b	A in B	ab	a b
Addition	plus	+	+
Subtraction	minus	□	_
Ratio	ad	pi	;
Square root , cubic root	VQ. VC.	V2 V3	R
Squares	Aquadratus, Aquad.	a2	a2
Cubes	Acubus, Acub.	a3	a3

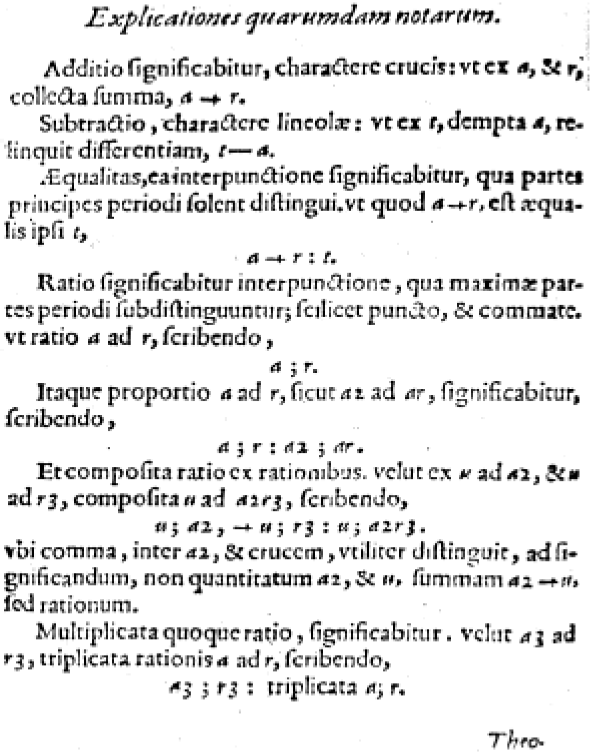
However, when Hérigone defines his notation in his section of *Algebra*, he identifies it with Viète's notation. As an example, we may refer to the explanation of the notation at the beginning of Algebra in the *Cursus,* where Hérigone presents his notation by transforming Viète's notation. For example, Hérigone writes "*ab* signifies *A* in *B*"; "*a2b* signifies *A* quadratum in *B*" and so on, the former being Hérigone's notation and the latter Viète's notation (see Figure 1).


Explicatio notarum, *des notes.*

ab	*signifi.*	A in B, *A multiplie par B.*
a2b		A quadratũ in B, *le quarré d'A multiplie par B.*
ab2		A in B quadratum, *A multiplie par B quarré.*
ap		A planum, *A plan.*
ap2		A plani quadratum, *le quarré du plan A.*
ap3		A plani cubus, *le cube d'A plan.*
a ſ		A ſolidum, *A ſolide.*
a ſ2		A ſolidi quadratum, *le quarré de A ſolide.*

Figure 1. Hérigone's introduction to Algebra (Hérigone, 1634, II, 5)

Twenty-five years later, Mengoli, at the beginning of his *Geometria* and on a separate page under the title *Explicationes quarundam notarum*, explains the basic notation he intends to use. Note that for representing the powers, Mengoli, like Hérigone, wrote the exponent to the right of the letter, as in Figure 2.



Explicationes quarumdam notarum.

Additio ſignificabitur, charactere crucis: vt ex *a*, & *r*, collecta ſumma, *a* + *r*.

Subtractio, charactere lineolæ: vt ex *t*, dempta *a*, relinquit differentiam, *t* — *a*.

Æqualitas, ea interpunctione ſignificabitur, qua partes principes periodi ſolent diſtingui. vt quod *a* + *r*, eſt æqualis ipſi *t*,

a + *r* : *t*.

Ratio ſignificabitur interpunctione, qua maximæ partes periodi ſubdiſtinguuntur; ſcilicet puncto, & commate. vt ratio *a* ad *r*, ſcribendo,

a ; *r*.

Itaque proportio *a* ad *r*, ſicut *a2* ad *ar*, ſignificabitur, ſcribendo,

a ; *r* : *a2* ; *ar*.

Et compoſita ratio ex rationibus. velut ex *u* ad *a2*, & *u* ad *r3*, compoſita *u* ad *a2r3*, ſcribendo,

u ; *a2*, + *u* ; *r3* : *u* ; *a2r3*.

vbi comma, inter *a2*, & crucem, vtiliter diſtinguit, ad ſignificandum, non quantitatum *a2*, & *u*, ſummam *a2* + *u*, ſed rationum.

Multiplicata quoque ratio, ſignificabitur. velut *a3* ad *r3*, triplicata rationis *a* ad *r*, ſcribendo,

a3 ; *r3* : triplicata *a*; *r*.

Theo-

Figure 2. Mengoli's notation in *Geometria* (Mengoli, 1659, I, 8)

We now proceed to analyze the development of the significance of the symbolic notation, specifically the significance of the letter representing a known quantity, which is different for the three authors. In fact, as we have said one of the major innovations in the late sixteenth century was the symbolic representation of the "given" by a letter in Viète's works.[16] This author uses separate symbols to represent both known and unknown quantities. Nevertheless, if we observe Viète's equation, we can appreciate the rhetorical form. We provide one example to show how Viète writes an equation:

"*B* in *A* – *A* quad. Aequatur *Z* quad" (Viète, 1970, 86)

which in modern notation would be written

$$Bx - x^2 = Z^2.$$

Certainly, for Viète this letter *B* represents the known quantity; that is, an indeterminate quantity but a given quantity. In 1591, he introduced the letter *B* to represent the known quantity; namely, the "given", although fixed, and its value can be arbitrarily selected. One may then speak about an indeterminate quantity being arbitrary, but a given quantity.

Later, Hérigone, when trying to adopt Viète's algebra in his *Cursus,* clearly legitimated this letter *B* as a kind or species of numbers, whose use in the algebraic rules does not depend on the value assigned. Let us show Hérigone's explanation of the status of the "given" letter:

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[16] On the analysis of the features of the development of symbolic language, see Serfati (2010, 108-111).

> Specious algebra is so-named from letters of the alphabet which have no particular meaning, either as discrete quantities, which are numbers, or as continuous quantities, except what one attributes to them. For example, if we attribute a value of 12 to the letter *B*, the reasoning applied to this letter *B*, without taking into account the number 12, applies to any other number, such as 15, 20, etc., and thus the letter *B* will represent these numbers as a kind, not as individuals or particulars. This must also be understood for continuous quantities, whether they be lines, surfaces or any other quantities one wishes.[17]

[17] L'Algèbre Spécieuse se nomme ainsi des lettres de l'alphabet, qui n'ont aucune signification particulier, ny en la quantité discrète, qui soit les nombres, ny en la continue, sinon celle qu'on leur attribue. Par exemple, si on attribue à la lettre B12 pour sa valeur, le raisonnement qu'on fera avec icelle lettre B, sans considérer le nombre 12, conviendra aussi à tout autre nombre comme à 15, 20, etc & par ainsi la lettre B signifiera l'espèce des nombres & non les individus & particuliers. Ce qu'il faut aussi entendre en la quantité continue, pouvant signifier une ligne, une superficie, ou autre quantité telle qu'on voudra. (Hérigone, 1642, VI, 76).

Hérigone goes on to explain this advantage for inventing universal theorems: “By means of these letters, one can invent universal theorems for both continuous and discrete quantities".[18]

Indeed, Hérigone in his *Algebra* tried to generalize some of Viète’s statements. The symbolic language in Hérigone’s hands allows obtaining new results. For instance, at the end of *De recognitione et emendatione aequationum, tractatus secundus* (1615) Viète gives examples of ambiguous Equations (equations with several roots) of degree 2, 3, 4, 5, but failed to provide a proof, claiming he had dealt with it elsewhere.

On the other hand, Hérigone states a theorem that generalizes this result. This theorem can be found at the end of Chapter 20 of Hérigone’s volume on *Algebra* (1634), after calculations (similar to those of Viète) that consisted in finding the upper or lower bounds in the numerical solutions of ambiguous equations. Hérigone, concludes by stating a theorem that generalizes his results:

[18] Par le moyen des quelles lettres on invente des théorèmes universels tant en la quantité continue que discrète (Hérigone, 1642, VI, 76).

If a positive power is affected by all possible lower degrees and by the independent term, which are alternately negative and positive, and the coefficient of the power following the highest power being the sum of as many numbers as there are unities in the exponent of the [highest] power; the coefficient of the following degree is the sum of all plane numbers of those numbers; the coefficient of the third degree is the sum of all solids, and so on as far as the independent term, which is the product of these numbers continuously multiplied; the number of all the positive terms will be equal to the number of all the negative terms and consequently if the independent term is on one side of the equation and the highest power and all lower degrees on the other side, the root of the equation may be expressed by each of the proposed numbers.[19]

[19] Si une puissance affirmée est affectée sous tous les degrés parodiques & sous l'homogène de comparaison, qu'ils soient alternativement niez & affirmez, & que le coefficient du degré parodique prochain à la puissance, soit l'agrégé d'autant de nombres qu'il y aura d'unités en l'exposant de la puissance : le coefficient du second degré inferieur suivant, soit l' agrégé de tous les plans des mêmes nombres : le coefficient du troisième degré, soit l'agrégé de tous les solides, & ainsi de suite jusques à l'homogène de comparaison qui est le produit

This theorem deals with finding ambiguous equations with a given set of roots, the importance of which Hérigone is at pains to stress. It can be stated in modern notation as:

$$x^n - a_{n-1}x^{n-1} + a_{n-2}x^{n-2} - \ldots + a_1 x = a_o$$

where if $p_1, p_2, p_3, \ldots p_n$ are n-roots of the equation, then

$$a_0 = \prod_{i=1}^{i=n} p_i \ ;$$

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des dits nombres multipliez continûment : la somme de tous les affirmez sera égale à la somme de tous les niez, & par conséquent si l'homogène de comparaison fait une partie de l'équation, & la puissance avec tous ses degrés parodiques l'autre partie, la racine de la puissance pourra être expliquée par un chacun des nombres proposez./ Si potestas affirmata, sit affecta sub omnibus gradibus parodicis, alternatim negatis & affirmatis, sitque coefficiens, primi gradus parodici à potestate, aggregatum totidem numerorum, quot sunt unitates in exponente potestatis : coefficiens secundi gradus, aggregatum omnium planorum eorundem numerorum : coefficiens tertij gradus, aggregatum omnium solidorum, & ita deinceps usque ad homogeneum comparationis, quod gignitur ex continua multiplicatione eorundem numerorum : aggregatum omnium affirmatorum erit aequale aggregato omnium negatorum, ac proinde si homogeneum comparationis faciat unam aequationis partem, & potestas cum omnibus suis gradibus parodicis alteram, radix potestatis erit explicabilis de quolibet illorum numerorum. (Hérigone, 1634, II, 195–196).

the other terms represent the sum of the roots p_i combined, that is,

$$a_{n-1} = \sum_{i=1}^{i=n} p_i \ ; \ a_{n-2} = \sum_{1 \le i < j \le n} p_i p_j \ ,$$

and so on.

In fact the analysis of the relationship between the roots of an equation and the coefficients of the equation constitutes a step forward into the development of the theory of algebraic equations in the 17th century.

Later, in 1659, in Italy, Mengoli probably the most original student of Cavalieri, developped a new and fruitful algebraic method for solving quadrature problems using Viète's and Hérigone's algebra. In fact, Mengoli having closely read Hérigone's *Cursus* introduced in his *Geometria* the new concept of a determinable indeterminate quantity. Mengoli's idea is that letters could represent not only given numbers, unknown or indeterminate quantities, but variables as well; that is, in Hérigone's words: determinable [but] indeterminate quantities. For instance, Mengoli constructs the summations in the *Elementum secundum* of his *Geometria* by a new means of writing and calculating finite summations of powers and products of powers. He did not give them values or wrote them using the sign + and suspension points (...), but rather created an innovative and useful symbolic construction that would allow him to calculate these summations (which he calls species), which he regarded as new algebraic expressions. He considered an arbitrary number or *tota*, represented by the letter *t*, and divided it into two parts, *a* (abscissa) and *r* = *t*-*a* (residua). He then took *tota* equal to 2, 3,... and gave examples up to 10. That is to say, if *t* is 2, *a* is 1, and *r* is 1. If *t* is 3, *a* may be 1 or 2 and *r* is then 2 or 1, respectively. He also calculated the squares and cubes of *a*, the products of *a* and *r*, of the squares of *a* and *r*, and so on. He then proceeded to add all the numbers *a* that he separated from the same number *t* . For instance, if *t* is 3, the summation will be 3, because it is the sum of 1 and 2; if *t* is 4, the summation will be 6, because it is the sum of 1, 2, and 3, and so on. He wrote *O.a* to express this sum from *a*=1 to *a*=*t*-1:[20]

$$O\,.\,a = \sum_{a=1}^{a=t-1} a$$

The summations that Mengoli obtains are indeterminate numbers, but they become determinate when we know the value of *t*. Mengoli describes the notation as follows:

[20] Obviously "*O.* " meant *Omnes* and originated with Cavalieri and his *Omnes lineae*.

> When I write *O.a.*,...you have the summation [*massa*] of all the abscissae: but what value this summation is you still do not know if I do not write what number the summation is. But if I assign *O.a.* to the summation of the number *t*, you still do not know how much it is if at the same time I do not assign the value of the letter *t*[21].

By assigning different values to *t*, Mengoli explicitly introduces the concept of the "variable", a notion that was rather new at the time. To clarify this idea, Mengoli adds:

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[21] Cum scripsero *O.a.* habes massam ex omnibus abscissi: sed quota sic haec massa, nondum habes, nisi scripsero cuius numeri sit massa. Quod si assignavero *O.a.*, numeri *t* massam esse; neque sic habes, quota sit, nisi simul assignavero, quotus est numerus, valor litterae *t* ...(Mengoli, 1659, 61).

> But when I allow you to fix a value for the letter *t*, and you, using this license, say that *t* is equal to 5, immediately you will accurately assign *O.a.* equal to 10, t^2 equal to 25, t^3 equal to 125, and *O.r.* equal to 10, and if the letters *t* are determinate, the quantities *O.a.*, O.r., t^2, t^3, will be determinate. Thus, before you have used the license given, you actually had *O.a., O.r.*, t^2, t^3,[which are] determinable [but] indeterminate quantities.[22]

It should be pointed out that Mengoli uses the "specious" language both as a means of expression and as an analytic tool. Therefore, Mengoli also applies his idea of "variable" to calculate the "quasi ratios" (nowadays, the limit) of these summations (Massa, 1997). The ratio between summations is also indeterminate, but becomes determinable by increasing the value of *t*. From this idea of quasi ratio, he constructs the theory of "quasi proportions" taking the Euclidean theory of

[22] Cum verò licentiam dedero, ut quotum quemque litterae *t* valorem taxes; tuque huiusmodi usus licentia dixeris, *t* valere quinario : statim profecto assignabis & *O.a.*, valere 10 ; & *t*2, valere 25 ;& *t*3, valere 125 ;& *O.r.*, valere 10 ;& determinatae litterae *t*, determinatas esse quantitates *O.a., O.r.,t*2, *t*3. Quare data licentia antequam usus fueris, habebas profecto *O.a., O.r., t*2, *t*3, quantitates indeterminatas determinabiles. (Mengoli, 1659, 61).

proportions as a model, which enables him to calculate the value of the limits of these summations.[23]

We have presented an example on the different strategies for representing the known by these authors, which in Mengoli's hands allows him to introduce the idea of variable and to develop the concept of limit. This provides us with a valuable example of the evolution for the understanding and the use of symbolic language through this process of transmission, appropriation and circulation in the seventeenth-century.

3. On Hérigone's symbolic method. From rhetorical explanations to symbolic lines

In order to clarify the role of the symbolic language we analyse the features of some proofs in the works under study. A study of the three texts reveals that the presentation of propositions is approached very differently, ranging from rhetorical explanations in Viète's works to

[23] This theory constitutes an essential episode in the use of the infinite and would prove to be a very successful tool in the study of Mengoli's quadratures and logarithms.

symbolic lines in Hérigone’s and Mengoli’s works. If we compare Hérigone’s presentation of the proof of an identity proposition with Viète’s similar identity, we can see that Viète gives rhetorical explanations and verbal descriptions, uses few symbols, employs capital letters to represent quantities, leaves no margins and writes the words “cubus”, “quadratus”, etc. to express powers. In contrast, Hérigone formulates all identities and properties, and even some statements in symbolic language, providing no rhetorical explanations or verbal descriptions, and he writes the powers using a number. Compare Figure 3 from Viète and Figure 4 from Hérigone.

PROPOSITIO XV.

CUBO adgregati duorum laterum, cubum differentiæ eorundem addere.

SIT latus unum A, alterum B. Oporteat A + B cubo, A = B cubum addere. At verò cubus effectus abs A + B, conſtat A cubo, + A quadrato in B ter, + A in B quadratum ter, + B cubo. Cubus autem abs A = B conſtat A cubo, — A quadrato in B ter, + A in B quadratum ter, — B cubo. Fiat igitur horum additio: ſumma eſt A cubus bis, + A in B quadratum ſexies. Hinc ordinatur

Figure 3. Proposition XV in Viète’s *Ad Logisticen Speciosam* (Viète, 1970, 20)

PROPOS. XIX.

Quadratum aggregati cuborum excedit quadratum differentiæ eorundem cuborum, quadruplo cubo rectanguli ſub lateribus.	*Le quarré de la ſomme de deux cubes excede le quarré de la difference des meſmes cubes, du quadruple du cube contenu ſous les coſtez.*

A, 3. B, 2.

Hypoth.

a & b *ſnt quantit.* D.

Req. π. demonſtr.

□.a3 ~+b3 2|2 □.a3~b3 ~+ 4a3b3,

Demonſtr.

1. d. 2	□.a3 ~+b3 *eſt* a6 ~+ 2a3b3 ~+ b6,	α
1. d. 2	□.a3~b3 *eſt* a6~2a3b3 ~+ b6,	β
concl. 18. a. 1	α~β *eſt* 4a3b3.	

Figure 4. Proposition V. XIX in Hérigone's Algebra (Hérigone, 1634, II, 46)

The most important aspect therefore is how Hérigone appropriates Viète's proofs and transforms these rhetorical explanations into a set of symbolic lines. In fact, Hérigone devises a new method using the symbolic language to present the proofs.

So, in the title he writes: "A Course of Mathematics demonstrated by a brief and clear new method through real and universal symbols, which are easily understood without the use of any language". Hérigone also claims that he had invented a brief and intelligible new method for making demonstrations; which he explains in the dedication “Au lecteur/ Ad lectorem” [To the reader]

> There is no doubt at all that the best method for teaching the sciences is that in which brevity is combined with ease. But it is not always easy to attain both, particularly in mathematics, which, as Cicero pointed out, is highly obscure. Having considered this myself, and seeing that the greatest difficulties are in the demonstrations, understanding of which depends on a knowledge of all parts of mathematics, I have devised a new method, both brief and clear, of making demonstrations, without the use of any language.[24]

[24] Car on ne doute point, que la meilleure méthode d’enseigner les sciences est celle, en laquelle la brièveté se trouve conjointe avec la facilité : mais il n’est pas aisé de pouvoir obtenir l’une & l’autre, principalement aux Mathématiques, lesquelles comme témoigne Ciceron, sont grandement obscures. Ce que considérant en moi-même, & voyant que les plus grandes difficultés estoites aux démonstrations, de l’intelligence desquelles dépend la connaissance de toutes les parties des Mathématiques, j’ai inventé une nouvelle méthode de faire les démonstrations, briefe & intelligible, sans l’usage d’aucune langue. /Nam extra controversiam est, optimam methodum tradendi scientias, esse eam, in qua brevitas perspicuitati coniungitur, sed utramque assequi hoc opus hic labor est, praesertim in Mathematis disciplinis, quae teste Cicerone, in

Moreover, Hérigone stresses the importance of knowing the symbols and understanding the proofs performed using this universal notation. We can distinguish three features in Hérigone's new method: the original notation, the axiomatic-deductive reasoning and the presentation of the propositions.

Concerning the first feature, throughout the book Hérigone uses an original notation with new symbols and new abbreviations to represent algebraic expressions, numbers and signs. In each volume of the *Cursus* he provides alphabetically ordered tables of abbreviations (which he calls « explicatio notarum »);[25] as in Figure 5.

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maxima versantur difficultae. Quae cum animo perpenderem, perspectum que haberem, difficultates quae in erudito Mathematicorum pulvere plus negotijs facessunt, consistere in demonstrationibus, ex quarum intelligentia Mathematicarum disciplinarum omnis omnino pendet cognitio: excogitavi novam methodum demonstrandi brevem, & citra ullius idiomatis usum intellectu facilem. (Hérigone, 1634, I, *Ad Lectorem*).

[25] It is noteworthy that Mengoli in his *Geometria* also provides a table of notation called « explicationes quarundam notarum », see Figure 2 in this article.

EXPLICATION DES NOTES.

~ minus, *moins.*

.~: differentia, *difference.*

⊰e inter ſe, *entr'elles.*

⊰n, in, *en.*

⊰ntr. inter, *entre.*

ıı, vel, *ou.*

π; ad, *à.*

5< pentagonum, *pentagone.*

6< hexagonum, *hexagone, &c.*

ν.4< latus quadrati, *le coſté d'vn quarré.*

ν.5< latus pentagoni, *le coſté d'vn pentagone.*

a2 A quadratum, *le quarré de A.*

a3 A cubus, *le cube de A.*

a4 A quadrato-quadratū, *le quarré-quarré de A.*

Et ſic infinitum, *& ainſi à l'infini.*

= parallela, *parallele.*

⊥ perpendicularis, *perpendiculaire.*

.. eſt nota genitiui, *ſignifie (de)*

; eſt nota numeri pluralis, *ſignifie le plurier.*

2|2 æqualis, *egale.*

3|2 maior, *plus grande.*

2|3 minor, *plus petite.*

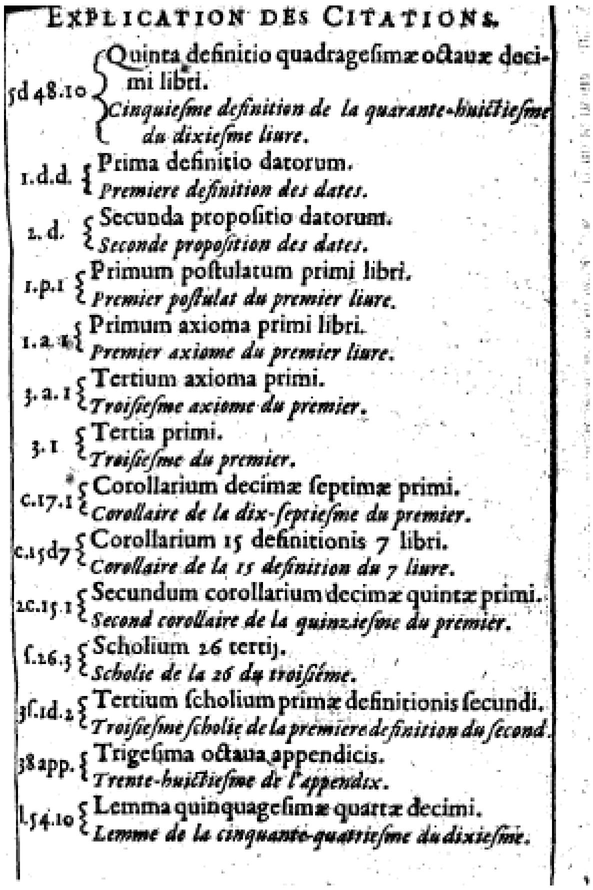
$\frac{1}{3}$ tertia pars, *le tiers.*

$\frac{1}{4}$ quarta pars, *le quart*

$\frac{2}{3}$ duæ tertiæ, *deux tiers.*

Figure 5. Hérigone's table of abbreviations (Hérigone, 1634, I, f. bv[r])

He also presents tables of explanations of the citations ("explicatio citationum") at the beginning of each of the volumes that make up the *Cursus*; (see Figure 6). The citations either refer to propositions in Euclid's *Elements* or to the *Cursus* itself. Thus, for example, "C.17.I" means "Corollarium decimae septimae primi. Corollaire de la dix-septième du premier".



EXPLICATION DES CITATIONS.

5d 48.10	Quinta definitio quadrageſimæ octauæ decimi libri. *Cinquieſme definition de la quarante-huictieſme du dixieſme liure.*
1.d.d.	Prima definitio datorum. *Premiere definition des dates.*
2. d.	Secunda propoſitio datorum. *Seconde propoſition des dates.*
1.p.1	Primum poſtulatum primi libri. *Premier poſtulat du premier liure.*
1. a. 1	Primum axioma primi libri. *Premier axiome du premier liure.*
3. a. 1	Tertium axioma primi. *Troiſieſme axiome du premier.*
3. 1	Tertia primi. *Troiſieſme du premier.*
c.17.1	Corollarium decimæ ſeptimæ primi. *Corollaire de la dix-ſeptieſme du premier.*
c.15d7	Corollarium 15 definitionis 7 libri. *Corollaire de la 15 definition du 7 liure.*
2c.15.1	Secundum corollarium decimæ quintæ primi. *Second corollaire de la quinzieſme du premier.*
ſ.26.3	Scholium 26 tertij. *Scholie de la 26 du troiſiéme.*
3ſ.1d.2	Tertium ſcholium primæ definitionis ſecundi. *Troiſieſme ſcholie de la premiere definition du ſecond.*
38app.	Trigeſima octaua appendicis. *Trente-huictieſme de l'appendix.*
l.54.10	Lemma quinquageſimæ quartæ decimi. *Lemme de la cinquante-quatrieſme du dixieſme.*

Figure 6. Hérigone's explanatory table of citacions (Hérigone, 1634, I, f. bvii^{r})

The second feature of Hérigone's method concerns his use of an axiomatic-deductive reasoning. Hérigone's originality resides not only in the explicit explanation of axiomatic-deductive reasoning, but also in its use in syllogisms, because in the demonstrations one can find in one symbolic line the major premise and the conclusion, using the former symbolic line as the minor premise. Hérigone's states this relation with syllogisms, as follows:

> And as each consequence depends immediately on the proposition cited, the demonstration is sustained from beginning to end by a continuous series of legitimate, necessary and immediate consequences.[26]

[26] En cette methode on ne dit rien qui n'aye esté expliqué & concedé aux premises... Et parce que chaque consequence depend immediatament de la proposition citée, la demonstration s'entretien depuis son commencement jusques à la conclusion, par une suite continue de consequences legitimes, necessaires & immediates / In hac methodo nihil adfertur, nisi fuerit in praemissis explicatum & concessum...Et quoniam singulae consequentiae ex propositionibus allegatis immediate pendent, demonstratio ab initio ad finem, serie continua, legitimaru, necessariarum que consecutionum immediatarum (Hérigone, 1634, I, *Ad Lectorem*).

> The demonstration is sustained from beginning to end by a continuous series of legitimate, necessary and immediate consequences, each one included in a short line, which can be solved by syllogisms, because in the proposition cited as well as in that corresponding to the citation one can find all parts of the syllogism.[27]

We now analyze the demonstration of Proposition I [28].

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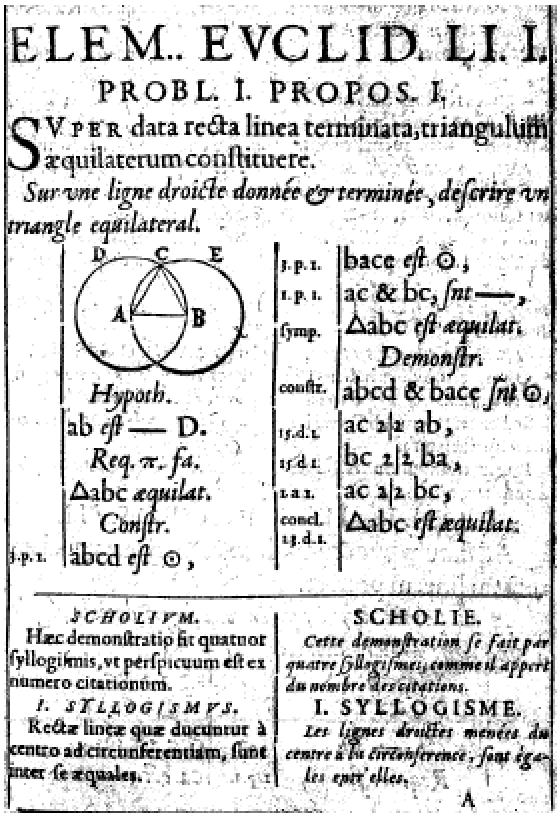
[27] La demonstration s'entretien depuis son commencement jusques à la conclusion, par une suite continue de consequences legitimes, necessaires & immediates, contenues chacune en une petite ligne, lesquelles se peuvent resoudre facilement en syllogismes, à cause qu'en la proposition citée, & en celle qui correspond à la citation, se trouvent toutes les parties du syllogisme: comme on peut voir en la premiere demonstration du premier livre, qui a esté reduite en syllogismes./demonstratio ab initio ad finem, serie continua, legitimarum, necessariarumque consecutionum immediatarum, singulis lineolis comprensarum aptè cohaeret: quarum unaquaeque nullo negotio in syllogismum potest converti, quòd in propositione citata, & in ea quae citationi respondet, omnes syllogismi partes reperiatur: ut videre est in prima libri primi demonstratione, quae in syllogismos est conversa. (Hérigone, 1634, I, *Ad Lectorem*).

[28] This demonstration is also found in our previous work where we analyzed how Hérigone reformulates Euclid's *Elements* from Clavius' *Elements* in symbolic language in his *Cursus*, see Massa, 2010.



ELEM.. EVCLID. LI. I.

PROBL. I. PROPOS. I.

SVPER data recta linea terminata, triangulum æquilaterum constituere.

Sur vne ligne droicte donnée & terminée, descrire vn triangle equilateral.

	Hypoth.	3.p.1.	bace *est* ⊙,
	ab *est* — D.	1.p.1.	ac & bc, *snt* —,
	Req. π. fa.	symp.	Δabc *est æquilat.*
	Δabc *æquilat.*		*Demonstr.*
	Constr.	constr.	abcd & bace *snt* ⊙,
3.p.1.	abcd *est* ⊙,	15.d.1.	ac 2|2 ab,
		15.d.1.	bc 2|2 ba,
		1.a.1.	ac 2|2 bc,
		concl. 23.d.1.	Δabc *est æquilat.*

SCHOLIVM.

Hæc demonstratio fit quatuor syllogismis, vt perspicuum est ex numero citationum.

I. SYLLOGISMVS.

Rectæ lineæ quæ ducuntur à centro ad circunferentiam, sunt inter se æquales.

SCHOLIE.

Cette demonstration se fait par quatre syllogismes, comme il appert du nombre des citations.

I. SYLLOGISME.

Les lignes droictes menées du centre à la circonference, sont egales entr'elles.

A

Figure 7. Proposition I.1 (Hérigone, 1634, I, 1)

In the first, book Hérigone proved the first proposition by his method and further by syllogisms and explains the identification of the premises in the demonstration. At the beginning, Hérigone states: "This demonstration is performed by four syllogisms, as one can perceive from the number of citations". He then explains all the syllogisms.

"III SYLLOGISM.

Those things that are equal to the same are equal to each other.

But the straight lines *AC* & *CB* are equal to the same straight line.

Therefore the straight lines *AC* & *CB* are equal to each other".[29]

So, in the III syllogism, Hérigone writes: "I. axiom. 1. *AC* = *BC*", the major premise is the first axiom of Euclid's book I, while the minor premise is deduced from the conclusions of the first and second syllogisms: *AC* = *AB* and *BC* = *BA*, and the conclusion of this third syllogism is *AC* = *BC*. These conclusions enable the minor premise in the last syllogism to be deduced.

"IV SYLLOGISM.

All triangles that have three equal sides are equilateral.

But the triangle ABC has three equal sides.

Therefore the triangle ABC is equilateral"[30].

[29] III. SYLLOGISME. Les choses égales à une mesme, sont égales entr'elles. Mais les lignes droites AC & CB sont égales à une mesme ligne droite. Donc les lignes droites AC & BC sont égales entr'elles./III. SYLLOGISMUS. Quae eidem aequalia sunt, inter se sunt aequalia. Sed rectae AC & BC sunt eidem rectae aequales. Igitur rectae AC & BC sunt inter se aequales. (Hérigone, 1634, I, 2).

In this case, Hérigone writes:" I. definition. 23. *ABC* is an equilateral triangle", the major premise is I.d.23, while the minor premise is deduced from the former conclusions *AC* = *AB*, *BC* = *BA* and *AC* = *BC*, and the conclusion of the third syllogism is that "the triangle ABC is equilateral", which concludes the demonstration.

Hérigone's originality resides not in the demonstration by using syllogisms, but rather in the identification of all parts of the syllogism as symbolic lines, which transforms the demonstration by syllogisms into another, shorter and easier one.

The third feature of Hérigone's method addresses the presentation of propositions. Hérigone divides his propositions into separate sections: hypothesis (known and unknown quantities), explanation or requirement, proof, and conclusion. In the margin he writes the number of propositions of Euclid's *Elements* that he is using. He occasionally gives the numerical solution (for example in an equation) in a section headed "Determinatio". In geometric constructions, he provides

[30] IV. SYLLOGISME. Tout triangle qui a trois costez égaux, est equilateral. Mais le triangle ABC a trois costez égaux. Donc le triangle ABC est equilateral. /IV. SYLLOGISMUS. Omne triangulum habens tria latera aequalia, est aequilaterum. Sed triangulum ABC tria habet aequalia latera. Igitur triangulum ABC est aequilaterum. (Hérigone, 1634, I, 2).

the instructions needed to make the drawing in a paragraph referred to as "Constructio". Hérigone writes as follows:

> The distinction of the proposition into its members, that is, the part in which the hypothesis is advanced, the explanation of the requirement, the construction or preparation and the demonstration, thereby relieves the memory and makes it very helpful for understanding the demonstration[31].

Indeed, it is important to point out that Hérigone sought to introduce a new, briefer and more intelligible method for making demonstrations in pure and mixed mathematics.

Now we wish to show how Hérigone's method, devised for a better understanding of Mathematics, was used by Mengoli for obtaining new results in his *Geometria* 25 years later. Like those by Hérigone, Mengoli's demonstrations are expressed in symbolic language

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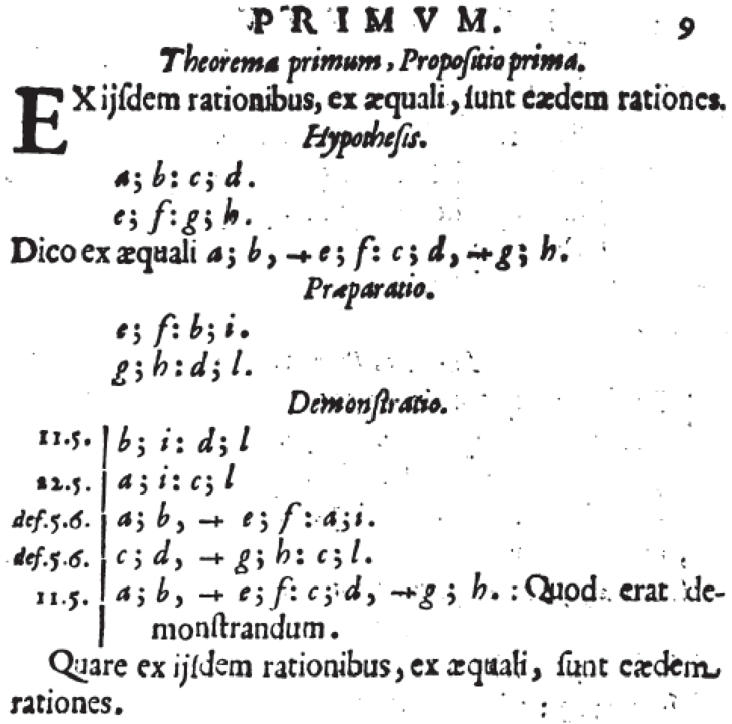
[31] La distinction de la proposition en ses membres, savoir en l'hypothese, l'explication du requis, la construction, ou preparation, & la demonstration, soulage aussi la memoire, & sert grandement à l'intelligence de la demonstration. / Praeterea distinctio propositionis in sua membra, scilicet in hypothesin, explicationem quaesiti, constructionem, vel praeparationem, & demonstrationem non parum iuvat quoque memoriam, & ad intelligendam demonstrationem multùm prodest. (Hérigone, 1634, I, *Ad lectorem*).

with logical statements consisting of a few lines. We can identify the syllogisms in the lines of this proposition (See Figure 8).



PRIMVM. 9

Theorema primum, Propositio prima.

EX ijsdem rationibus, ex æquali, sunt eædem rationes.

Hypothesis.

a; b: c; d.

e; f: g; h.

Dico ex æquali *a; b, + e; f: c; d, + g; h.*

Præparatio.

e; f: b; i.

g; h: d; l.

Demonstratio.

11.5. | *b; i: d; l*

22.5. | *a; i: c; l*

def.5.6. | *a; b, + e; f: a; i.*

def.5.6. | *c; d, + g; h: c; l.*

11.5. | *a; b, + e; f: c; d, + g; h.* : Quod erat demonstrandum.

Quare ex ijsdem rationibus, ex æquali, sunt eædem rationes.

Figure 8. Proposition I.1 in Mengoli's *Geometria* (Mengoli, 1659, I, 9)

Mengoli's goal was to create a new discipline, a specious geometry modelled on Viète's specious algebra, by further developing Hérigone's symbolic language. He refers to Euclid's *Elements* using conventions similar to those of Hérigone. For instance, Mengoli writes "22.5" in the margin to indicate his use of Euclid V.22 . In the proof he writes *a* ; *i* : *c* ;

l" (modern notation: $a : i = c : l$). In his *Algebra*, Hérigone had written "22.5" in the margin and " *ik* ∏ *m* 2/2 *fd* ∏ *de*" (modern notation: $ik : m = fd : de$) in the proof (Hérigone, 1634, II, 148).

A further important relation lies in the demonstrations and in the presentation of the propositions. Mengoli writes all his proofs in Hérigonean style. Mengoli's propositions, like those by Hérigone, are divided into parts, such as "Hypothesis", "Praeparatio" and "Demonstratio". Mengoli, who was influenced by Hérigone's idea of symbolic language as a powerful tool, also absorbed his method of presenting demonstrations.

The role of symbolic language in Mengoli's *Geometria* is both significant and original. In fact, the arithmetic manipulation of these algebraic expressions helped Mengoli to obtain new results, he derived unknown values for the areas of a large class of geometric figures at once, and new procedures like the summations, the rules of sum of *k*th-powers of *th*-integers, etc.

4. Conclusion

We have described a brief episode from the process of algebraization of mathematics which took place gradually and in very different ways in several locations during the early seventeenth century. We must emphasize that Hérigone presents an original symbolic language as a universal language for working with pure mathematics as well as mixed mathematics. In fact, his project constitutes a new method that enables him reformulate known mathematics in a symbolic language, such as from Viète's work, Euclid's works and others. This symbolic language allows expressing mathematics in short lines and renders the demonstrations briefer, clearer and, as Hérigone remarked, relieves the memory.

Regarding the circulation of these ideas, not only is it important for one author or another to use the same symbols to represent quantities (powers); more important is the significance of these symbols for representing any magnitude (discrete or continuous) throughout the process of reasoning in the demonstrations or for the resolution of the problems. Certainly, Viète introduces his logistical "speciosa" for dealing with any magnitude; Hérigone for his part wishes to introduce this universal language for teaching and validating both pure and mixed mathematics providing universal theorems, while Mengoli finally uses symbolic language for finding new results and for creating a new discipline in mathematics as a "geometry of species".

Through the reception of Viète's statements and rhetorical demonstrations, Hérigone introduces a new symbolic language and a new method of axiomatic-deductive reasoning for improving the understanding of Viète`s rhetorical demonstrations and of all pure and mixed mathematical demonstrations. This symbolic language is expressed in short lines following a logical structure which can be identified by syllogisms. Moreover, the divisions established in the demonstrations make Hérigone's demonstrations clearer than Viète's rhetorical demonstrations, and enable all the steps in the process to be seen at once. Hérigone followed up on Viète's analytic art by introducing this new method for a better understanding of the results in mathematical demonstrations. The justification for the use of this method adopting the symbolic language therefore resides in its didactical purpose.

Mengoli, who read Hérigone's *Cursus*, subsequently absorbed Hérigone's ideas in his *Geometria*, and presented his demonstrations using Hérigone's procedure, thereby enabling him to arrive at new results.

In conclusion, this new method of demonstration using a universal language and logical sentences by means of axiomatic-deductive reasoning is absolutely original, and provides us with an insight into clarity of the logical structure of both Hérigone's and Mengoli's thinking.

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